

Rehabilitation of Concepts

Pavel Materna

materna@lorien.site.cas.cz

I. Introduction

Jerry A. Fodor in his [1998] writes:

”My topic is what concepts are.”

This is not my topic. In my opinion, any question of the kind “What are concepts?” is a futile question. My argument goes as follows:

The *expression* “concept” (as well as its translations like “pojem”, “Begriff” etc.) is homonymous. The way it is used and understood is essentially influenced by the context; we could speak about *concepts in the sense of psychology*, *concepts in the sense of logic*, *concepts in the sense of metaphysics*, etc. Before to say one word about concepts we have to specify the *sense* in which we intend to use the term. Thus our task is to formulate an *explication* in the Carnapian sense.

The context I am interested in here is a context of *logic*, in particular of what can be called *logical analysis of natural language* (“LANL”). In Section II the main consequence of this decision is articulated. Section III critically analyses the *set-theoretical* theories of concepts, in particular the Fregean conception as formulated in his [1891, 1892]; also Kauppi’s attempt (see [Kauppi 1967]) is mentioned. A general doubt concerning any set-theoretical theory of concepts is formulated in this section. Section IV appreciates Church’s generalisation of the notion of concept and shows that on this generalisation LANL can be considered to be a theory of concepts. The problem of hyperintensionality (Carnap, Cresswell, Bealer) is analysed in Section V, where Bolzano’s ingenious suggestion in his [1837] is emphasised. The final Section VI brings a positive proposal based on Tichý’s *transparent intensional logic (TIL)*, see, e.g., [Tichý 1988], and [Materna 1998]. The theory proper is suggested and some results referred to.

II. Concepts as non-mental entities

In the traditional textbooks of logic the chapter *Concept* made it clear that concepts were – as ‘reflections of essential properties of an object’ – a kind of images (representations, *Vorstellungen*) in the psychological sense of the word. (See, e.g., [Ziehen 1920].) As the

process of ‘depsychologisation’ of logic (connected with the names Bolzano, Frege, Husserl) won its battle the contemporary logic has lost any interest in classifying CONCEPT with logical terms. On the other hand, it was just Bolzano, Frege and other more recent thinkers who saved considerations concerning concepts for logic. This endeavour can be justified only if concepts as what can be denoted by the word “concept” get rid of mental character. This ‘purification’ (properly speaking an explication) of the notion CONCEPT did not result in a systematic introducing of the term *concept* into the textbooks of modern logic (symbolic as well as philosophical logic): the structure of a typical modern textbook of logic does not involve the category of concepts. All the same, such names as Church, Kauppi, Bealer, Peacocke and others, suggest that contemporary logic and analytic philosophy is aware of the importance of this category. So when can logicians and analytic philosophers accept concepts as entities that can be rationally discussed? It is by now clear that the necessary condition thereof is – at least for logicians – that concepts are declared to be abstract, non-mental entities. As Peacocke says:

Concepts are abstract objects.

([Peacocke 1999, 99])

or

If ‘meaning’ is used correlatively with ‘sense’, meanings are the concepts expressed, *not the mental representations of them*. (Emphasis mine.)

(*ibidem*, 237)

True, some analytic philosophers may not accept this non-mental character of concepts (Fodor, Jackendoff, Bartsch) but their concepts can be re-interpreted as mental representations of non-mental concepts. We set these variations aside. As for logicians, there is no compromise possible: if concepts are to be handled by logic then they simply cannot be mental entities. (The same holds for LANL.)

III. Set-theoretical entities?

Confining ourselves to 19th and 20th centuries we have to register first of all two great logicians who explicitly handled concepts without sharing psychologistic prejudices of their time: Bolzano and Frege. There are special reasons for talking about Bolzano later, against the time flow. I would like to stop now at Frege’s theory, which is a paradigmatic example of what I call ‘set-theoretical conception of concepts’ (see my [1998, *passim*]).

Summarising Frege's theory, as it is known from his [1891] and [1892] we can describe his idea as follows:

1. Concepts are functions that associate with every object **A** a truth-value: TRUE if **A** falls under the concept, FALSE otherwise.
2. Concepts are *denoted* (!) by 'concept words' (*Begriffsworte*).
3. As functions concepts are 'unsaturated'.
4. Because of 3. a concept word cannot stand in the subject position of a sentence.

Now the point 1. as in a nutshell characterises Frege's set-theoretical conception. We will try to show that the set-theoretical approach is incompatible with most intuitions we have using the term *concept* in the contexts of logic and LANL.

- A.** Concepts-functions according to 1. are nothing other than *characteristic functions of classes*. This means, among other things, that Fregean concepts are always *universalia*, i.e., Frege does not accept *individual concepts*. (Indeed, this also follows from his letting names and descriptions denote *objects*, '*Gegenstände*', which are *ex definitione* distinct from concepts.) Yet there are many concepts that we are ready to call *individual concepts*, such as THE SQUARE ROOT, THE LEAST PRIME, THE POPE, etc. Besides, there are also concepts to which various kinds of *emptiness* can be ascribed.
- B.** In the case of mathematical concepts the point 1. (confirmed by 2.) does not allow us to distinguish between the concept and the respective object. According to Frege the expression *prime number* denotes the (characteristic function of the) class of prime numbers; at the same time it denotes the concept PRIME NUMBER. But we would certainly like to make a distinction between the two. For example, we would like to see the *concept* PRIME NUMBER as something what determines the respective class. And one of the consequences of the points 1. and 2. is that there would be no two distinct concepts EQUIANGULAR TRIANGLE and EQUILATERAL TRIANGLE, since what is denoted by the respective expressions is one and the same class. In general, there could be no distinct concepts that would determine one and the same object. (To this point, a key point of our criticism, we will return in Section V.)
- C.** Another unpleasant consequence of Frege's theory concerns the case when the concept is an empirical one. If concepts are essentially classes, then we get a radical extensionalism, incompatible with the nature of empirical expressions/concepts: any empirical concept would lose its identity, since with any

change of the ‘population’ of the objects that fall under the concept we would have another class, and, therefore, another concept. (This phenomenon of *temporal variability*, as well as a similar but not as transparent phenomenon of *modal variability*, has been taken into account as soon as P(ossible-)W(orld) Semantics have been able to define intensions; the latter are also defined as set-theoretical objects but their incompatibility with some important intuitions is not as striking as it is in the case of Fregean concepts-classes. We will return to this point in the following sections.)

- D.** The points 3. and 4. are specific, being characteristic of Frege’s philosophy. The claim that functions are unsaturated is not very clear. True, having a Fregean *concept word*, say, ‘(is a) dog’, we do not claim anything, no sentence is present, and the latter arises till after a subject word is added (say, ‘Bessy’). But then why the names or descriptions are not said to denote unsaturated objects (‘Gegenstände’)? Does the expression like ‘Bessy’ or ‘the capital city of Poland’ claim something? A very weak point in Frege’s philosophy. As for the point 4., the well-known polemic Frege – Kerry helps us understand that the notion of unsaturatedness is not a useful notion. Frege’s claim that as soon as a concept word stands on the position of a subject word it is no more a name of a concept is absurd: why should a concept lose its identity when something is predicated about it? (We will see in the final section that the problem can get an excellent solution if Fregean position is abandoned.)

It seems to me that the Fins ([Kauppi 1967] and [Palomäki 1994]) have built up a modern version of the set-theoretical conception. Kauppi is well aware of the problems connected with the traditional doctrine; she exploits the means offered by the modern logic and formulates a nice theory that tries to solve the problems concerning the relation between ‘intension’ and ‘extension’ of a concept. Nevertheless, her (intensional) *containment* relation is, on the one hand, primitive, undefined in her system, but, on the other hand, it is pre-theoretically explained in such a manner that it corresponds to the traditional definition of the relation between ‘intension’ (‘content’) and ‘extension’ of a concept. There are many interesting results in Kauppi’s monograph but her theory is still a set-theoretical one.

Well, some attempts may try to distinguish between concepts as sets and concepts as ‘non-sets’ (Schock’s ‘attributes’). Yet the nature of ‘non-set-theoretical concepts’ is completely unclear.

To summarise, my claim is that any set-theoretical conception of concepts necessarily lays itself open to at least the points **A.** and **B.** above. If concepts are defined as intensions in the sense of PWS, then the point **C.** does not apply but then we have no mathematical concepts. In general, it holds what Zalta has formulated in his [1988]:

Although sets may be useful for describing certain structural relationships, they are not the kind of thing that would help us to understand the nature of presentation. There is nothing about a set in virtue of which it may be said to present something to us.

IV. Concepts and meanings. Church

As we will see in the next section, for Bolzano any expression with the exception of sentences can be connected with a concept. As we have seen in Section III, Frege acknowledges only general expressions like common nouns as denoting (!) concepts. A most universal proposal in this respect can be found in [Church 1956]. Church was essentially a follower of Frege but he disliked Frege's definition of concepts (and rightly so, as we have argued in the preceding section). Church instead of Frege's conception of concepts exploited Frege's idea well-known from [Frege 1892a]: the idea that between an expression and the object denoted a 'mode of presentation' (*die Art des Gegebenseins*) stands, called 'sense' (*Sinn*) by Frege. Frege's sense should be a path that links the expression with the object denoted. To Church, the same can be claimed about a concept. Thus Church proposes the following schema:

*an expression expresses its sense = a concept **of** its denotation*

An important consequence thereof is that *every meaningful expression is connected with a concept*, in particular also *sentences express concepts*.

One could object that concepts are traditionally distinguished from what a sentence can express since sentences – unlike other parts of language – can be true or false. This objection is easily refutable as soon as we closer analyse the two possible cases:

- 1) *A mathematical sentence* can be said – as known from Frege – to denote a truth value. It would have to express a concept of that truth value if concepts were considered to be senses (meanings) of expressions (and so Frege's 'thoughts' (*Gedanken*) in the case of sentences). But would it mean that *this concept itself* were true or false? Rather it would be just something like a 'path' to the truth value.

- 2) Still more obvious this is in the case of *empirical sentences*. Notwithstanding Frege's opinion that all (and thus also empirical) sentences denote truth values we can see today that empirical sentences denote at most truth-conditions, i.e., what PWS defines as *propositions*. Hence their meanings/ concepts determine the latter; a concept expressed by an empirical sentence identifies the truth-conditions of the latter. Possessing such a concept we *understand which proposition is given, we do not know in virtue of this concept alone whether this proposition is true or false*. In other words, identification is not the same as verification.

Church's generalisation of the notion of concept is an important step towards creating such a theory of concepts which would respect and exploit the means of the contemporary logic and semantics: *if concepts are meanings, then a contemporary theory of concepts is a kind of articulating semantics of natural language (or of LANL, if you like)*.

Remark: Concepts as meanings are, of course, abstract. Thus it is not a precise formulation to say that they *are* meanings. A meaning is always a meaning *of an expression*. To identify concepts with meanings presupposes that before the respective expression has been introduced into the given language there was no 'respective' concept, i.e., that concepts *come into being* dependently on introducing expressions into a language. But what *comes into being* is no more abstract: it can be – at least temporally – localised. Thus our conception is a Bolzanian one: concepts can *become* meanings of expressions, i.e., they can be 'attached' to expressions but they are definable independently of language. –

V. Bolzano and hyperintensionality

In [Bolzano 1837] we find an excellent theory of concepts incomparable with any such theory of that time. Bolzano, as a realist and an enemy of psychologism, developed a consistent conception of abstract, language independent concepts. Having defined 'sentences as such' (Sas, *Sätze an sich*) as abstract entities which are shared by 'real sentences' if written, thought or spoken, he considers the former as structured; those parts of Sas that are no Sas themselves Bolzano calls 'images as such' (*Vorstellungen an sich*) and the most important kind of the latter are just 'concepts' (*Begriffe*). Concepts also are, in general, structured, being composed of other concepts. Now on p.244 we find a small remark that indicates Bolzano's ingenious insight. Analysing the traditional problem of the relation between the 'content' ('intension', *Inhalt*) and 'extension' of a concept Bolzano adduces examples of concepts that share the content and are distinct. One such pair is

an unlearned son of a learned father, a learned son of an unlearned father.

Bolzano takes the content of a concept to be a sum (we would say ‘set’) of (obviously simple) concepts that the original concept is composed of (an essentially more general conception than the traditional one where the ‘subconcepts’ had to behave as composing the original concept *conjunctively*). Thus the content of both the concepts above would be (in a modern jargon)

{un/not, son-father, learned}

Since the concepts are distinct, Bolzano explains the distinction between a concept and its content as follows:

*Da unter diesem Inhalte nur die **Summe** der Bestandtheile, aus denen die Vorstellung besteht, nicht aber die **Art, wie** diese Theile untereinander verbunden sind, verstanden wird: so wird durch die blosse Angabe ihres Inhaltes eine Vorstellung noch nicht ganz bestimmt...*

What does it mean? What Bolzano says is a much more important idea than he himself perhaps thought (and naturally, it has been totally forgotten in the following decades).

Summing up we can ‘translate’ Bolzano as follows:

*Whereas the particular members of the content of a concept make up a set the concept itself is the **way** in which these members are connected.*

So a concept is not reducible to a set (*Summe*): it is rather a way of composing particular members of its content. But this means farewell to set-theoretical definitions of concepts.

Independently of Bolzano some later thinkers became aware of some problems connected with set-theoretical theories of concepts (mostly using ‘meaning’ instead of ‘concept’). In the first place we have to name Carnap because of his recognition in [Carnap 1947] of the fact that in some contexts (in particular, propositional attitudes) his method of intensions and extensions does not work and that meaning should in some way or other correspond to the *structure* of the respective expression. (This is why Carnap has proposed his ‘intensional isomorphism’.) The idea of structuredness has been explicitly formulated in [Cresswell 1975] where Cresswell speaks about *hyperintensionality* as a higher phase of intensionality, and in [Cresswell 1985]: *meanings have to be structured*. This story in the case of ‘structured propositions’ is competently described in [King 2001] (including Russell’s old conception of propositions).

But Bolzano’s remark as quoted above makes it clear that Bolzano knew or at least suspected that concept (meaning) had to be construed as being a *way* to the object; it is

however way – unlike the goal of a journey – what is structured. A good simile in this respect can be found in [Tichý 1988,p.1]:

When one travels from Los Angeles to New York, going, say, by way of St.Louis, Chicago, and St.Louis again, one's *destination* and the *itinerary* one follows to get there are clearly two distinct items. There is no sense in which Los Angeles, St.Louis, or Chicago are parts, or constituents, of New York. Each of the three cities, on the other hand, is an inalienable constituent of the circuitous itinerary in question, and the removal of any of them produces a different itinerary. An itinerary is a compound in which a number of locations occur, some of them possibly more than once, as St.Louis does in our example.

True, the resemblance of this simile to Bolzano's 'way of combining' is not absolute: the former illustrates the fact that distinct concepts ('itineraries') may lead to one and the same object ('destination') whereas the latter shows that distinct concepts may share the same components. Yet as for the 'itinerary example' it can be changed so that Bolzano's suggestion is preserved: An itinerary beginning in Chicago and going over St.Louis and New York to Los Angeles is distinct from the previous one, leads to another destination, but its components (the respective towns) are the same. On the other hand, Bolzano knew very well that two distinct concepts may lead to the same object, see his example with two concepts of triangle, one in terms of having three sides, the other in terms of having the sum of its angles equal to $2R$. (By the way, the fact that neither in the middle of the 20th century Bolzano's fine-grained definitions have not been understood is well documented by Bar-Hillel's criticism of this Bolzano's distinguishing between two distinct concepts of one and the same set: see [Bar-Hillel 1950])

The question whether there can be two distinct concepts that determine one and the same object is a paradigmatic question: the positive answer is incompatible with the set-theoretical doctrine. It is interesting to compare two standpoints to this question as they have been articulated by two practically contemporary thinkers: K.Gödel and G.Bealer.

In [Gödel 1990] Gödel compares his conception of concepts with the intuitionistic one (he calls the intuitionistic or better constructivistic concepts 'notions') and says:

any two different definitions of the form $\alpha(x) = \varphi(x)$ can be assumed to define two different notions α in the constructivistic sense. (L.c. p.128)

In contrast Gödel sees concepts as

the properties and relations of things existing independently of our definitions and constructions. (*Ibidem*)

Gödel's opposition to constructivists can be probably at least partially explained in terms of philosophical standpoints: as a Platonist he could not accept constructivists' *subjective* constructions as determining concepts: the latter had to be independent of minds.

A Platonist could however react in another way. The category of *constructions* (say, *abstract procedures*) need not be conceived as to be subjective. *As soon as constructions are defined as mind and language independent entities we can accept that various distinct constructions of one and the same object are good candidates for being distinct concepts of one and the same object: concepts defined in this way remain being objective and therefore acceptable by a Platonist.*

Gödel's concepts are, on the other hand very vague: talking about properties and relations in the area of mathematics cannot mean anything other than talking about classes and relations-in-extension, which can be criticised in the same way as we have done referring to Frege.

The view that distinct definitions determine distinct concepts has been convincingly articulated by George Bealer in his [1982]. Bealer distinguishes two kinds of "intensions": the first kind involves 'qualities' (non-Goodmanian properties), 'connections' (i.e., relations) and 'conditions' (evidently what is handled by PWSs as 'propositions') whereas the second kind contains 'concepts' and 'thoughts'. Bealer tries to define the distinction between the two kinds in an axiomatic way; that the 'intensions' of the second kind are more *fine-grained* than those of the first kind is however clear: a thought is given by just one 'logical tree' using 'thought-building operations' whereas infinitely many trees using condition-building operations determine one and the same condition. Further: the 'intensions of the first kind' are sufficient for analysing modal contexts but insufficient for analysing attitudinal contexts, where the 'intensions of the second kind' are "ideally suited for the treatment of intentional matters' (p.4). (This is a renaissance of Carnap's problem!) Thus for Bealer two distinct definitions determine two distinct concepts, even if the object that falls under these concepts is the same. So Bealer would in this respect take side with intuitionists against Gödel although he is no intuitionist.

VI. Concepts as constructions

In [Materna 1998] I have formulated a theory of concepts that would construe concepts as being *non-mental abstract procedures*. For logically handling such procedures it

was necessary to use or create some general conception. The best conception in this respect I know is Tichý's *transparent intensional logic* (TIL), see, e.g., [Tichý 1988]. Naturally, I cannot expound TIL here, so I will only articulate – as informally as possible – some important points.

1. Simple hierarchy of types

Concepts are considered to be *abstract procedures that aim at identifying objects*. Thus the first task consists in determining the area of objects that a natural language can talk about. First of all we can state that with the possible exception of proper names our empirical (as well as non-empirical) expressions never denote concrete, spatio-temporally localisable objects. We can at most talk about properties, relations-in-intension, propositions etc.. It could seem, of course, that talking about the highest mountain we talk about Mount Everest, but actually we use the term “the highest mountain” not to denote Mount Everest but to name the condition (‘office’ is Tichý’s term, we often use ‘role’) an individual has to fulfil to be the highest mountain. Similarly when we say that some dogs are dangerous we do not talk about some *class* of concrete dogs and about the *class* of dangerous objects: all what is denoted by these two terms is the *property being a dog* and the *property being dangerous*. We claim (saying “Some dogs are dangerous”) that there are some individuals that possess both the former and the latter property. Nothing is claimed about concrete individuals. (This is what already Frege in his [1884] has emphasised.) In general it holds that empirical expressions denote (and thus empirical concepts identify) *intensions* whereas non-empirical (in particular, logical and mathematical) expressions denote *extensions*. In TIL intensions are conceived of similarly as in the standard P(ossible-)W(orld)S(emantic)s: as functions the domain of which is the set of *possible worlds*. More precisely, intensions are functions that associate every possible world with a *chronology* of some kind of object. *Extensions* are then simply non-intensions.

The area of objects we can talk about is thus an area of *abstract* objects. The various relevant kinds of these objects can be handled in terms of a *hierarchy of types*, first, a simple one, later a ramified one.

The simple hierarchy is based on four atomic types. The complex types are defined *functionally*, as sets of partial functions from arguments of types β_1, \dots, β_m to values of type α (written as $(\alpha\beta_1 \dots \beta_m)$).

The atomic types are:
 truth-values, $\{\mathbf{T}, \mathbf{F}\}$, type \circ (omikron)
 universe, individuals, type ι
 time points (= real numbers), type τ
 logical space, possible worlds, type ω .

The types \circ, ι correspond approximatively to Montague's t, e , respectively. TIL is what Gallin in [1975] calls "two-sorted" system: ω is a separate type (unlike Montague's s) and the variables ranging over ω (as well as the variables ranging over τ) are explicitly used.

Now we show some examples of associating types with kinds of object that can be denoted by expressions (and so identified by concepts):

Classes of objects of a type α are identified with their characteristic functions, so their type schema is $(\circ\alpha)$. In particular, $(\circ\iota)$ is the type of classes of individuals, $(\circ\tau)$ is the type of classes of (real) numbers or of classes of time points, $(\circ(\circ\tau))$ is the type of classes of classes of numbers/ time points.

Relations (-in-extension) of objects of types β_1, \dots, β_m : again their characteristic functions, type schema $(\circ\beta_1\dots\beta_m)$.

Truth functions: unary $(\circ\circ)$, binary $(\circ\circ\circ)$.

Quantifiers: for any type α they may be considered to belong to the type $(\circ(\circ\alpha))$, or to the type $((\circ(\circ\alpha))(\circ\alpha))$.

All these examples are examples of *extensions*. Now some interesting intensions.

In general, an intension is a function of the type $((\alpha\tau)\omega)$ for a type α . Abbreviating this notation as $\alpha_{\tau\omega}$ we have:

Propositions, type $\circ_{\tau\omega}$

Individual roles/offices, e.g., the highest mountain, type $\iota_{\tau\omega}$

Properties of objects of type α , e.g., (being a) mountain, type $(\circ\alpha)_{\tau\omega}$

Relations-in-intension of objects of types β_1, \dots, β_m , e.g., taller than, believe, type $(\circ\beta_1\dots\beta_m)_{\tau\omega}$

Magnitudes, e.g., the number of planets, type $\tau_{\tau\omega}$

etc. etc.

If an intension returns the same value in all pairs $\langle w_i, t_j \rangle$, then it is called *trivial*.

Sometimes we are told that the claim *Empirical expressions denote just intensions* is incompatible with the way we speak: Saying, e.g., that Mount Everest is a mountain we surely

do not claim that Mount Everest is a property. Such objections are easily refutable. That Mount Everest is a mountain means that Mount Everest *possesses* the property **being a mountain**. Similarly to say that the highest mountain is in Asia means to say that whatever plays the role of the highest mountain possesses the property **being in Asia**. In this second case no proper name of Mount Everest is used, so that the sentence does not speak about Mount Everest at all. Its expressive power is independent of which object happens to be the highest mountain (in the given possible world at the given time point).

These considerations are characteristic of TIL as an intensionalistic system. They make it possible to resolve well-known semantic puzzles and to distinguish between empirical and non-empirical concepts: *empirical concepts identify non-trivial intensions*.

Here we cannot argue justifying particular features of TIL. We will only show how a theory of concepts as of not set-theoretical entities can be formulated based on TIL. To fulfil this task we have to define the core notion of TIL, viz. *constructions*.

2. Constructions

We have seen that Bolzano's concepts were *ways of combining* the elements of their contents. In my wording, concepts are *abstract procedures*. Church's λ -calculus, in particular the typed one, is an ideal tool for handling functions and their combining. Not surprisingly then TIL exploited the ideas of the typed λ -calculus (similarly as Montague) to define constructions. A brief informal description of the theory of constructions will be helpful, although no exact definitions can be adduced. As a result we will be able to say what a concept is from such a procedural viewpoint as is represented by TIL.

The first point to be emphasised is: *Constructions are distinct from expressions*. In other words: constructions are no expressions of some artificial language. (So they are no λ -terms either.) The way we will denote them, i.e., our notation does not justify us to identify this notation with the constructions themselves. This principle (frequently not understood or at least underestimated by symbolic logicians) is in good accordance with Church's idea that concepts should stand in the place of Frege's *senses* (see Ch.IV.), since Frege's senses were intended to link an expression with the object denoted.

Second, the most elementary, 'atomic' (and in a sense 'incomplete') constructions are *variables*. This seems to contradict the preceding claim: we are used to construe variables as being letters, characters, i.e., parts of a language. For TIL, such letters are only *names* of variables: variables themselves are a kind of construction – they construct objects dependently on *valuation*, i.e., a total function that associates each variable of objects of a type α with just

one member of α . Thus we say that variables *v-construct* objects, v being the parameter of valuations.

Third, every theory of procedures has to end somewhere, to have a stop. The first complete kind of construction in TIL is such a stop. It is called *trivialisation* and denoted by 0X , where X is an arbitrary object (including constructions). Trivialisation takes the object X and returns it without any change – it is an ‘immediate’ construction of X . In spite of a seeming banality it is an extremely important construction; its value can be appreciated within the ramified hierarchy of types (see below).

Fourth, the remaining two kinds of construction are – at least in their linguistic version – well-known from λ -calculi.

Where X, X_1, \dots, X_m are constructions *v-constructing* respectively objects of type $(\alpha\beta_1 \dots \beta_m)$, β_1, \dots, β_m , $[XX_1 \dots X_m]$ is a *construction* called *composition* that *v-constructs* the value (if any!) of the function *v-constructed* by X on arguments *v-constructed* by X_1, \dots, X_m . Compositions can be (*v-*)*improper*, i.e., they can *v-construct* nothing due to the partiality of respective functions.

Finally, the *closure*, $[\lambda x_1 \dots x_m X]$, also ‘abstraction’, creates a function in the well-known way.

Notice that neither compositions nor closures contain brackets and that closures do not contain, e.g., λ : these are only our means of notationally fixing the constructions, the latter are a kind of abstract procedure fully independent of the way we fix them linguistically.

The choice of kinds of construction is, of course, one of possible options. What makes it most attractive (and what explains why λ -calculus has inspired it) is that all thinkable procedures are reducible to the two fundamental ones: *creating a function* and *applying a function to its arguments*. (Variables and trivialisation are, of course, necessary as ‘interface’ between operations and the area of their applicability.)

Within the simple hierarchy of types it is impossible to ascribe types to constructions themselves. Defining a *ramified hierarchy* we get the possibility of handling constructions as objects *sui generis*. We will not reproduce the definition in question, suffice it that *first*, constructions of order n are defined, and the set of all constructions of order n , denoted $*_n$, is the type of order $n + 1$.

Example: Let x be a numerical variable ranging over τ . Then the order of the construction x is 1, so that the type of x is $*_1$ and its type is of order 2. 0x , i.e., the construction that constructs x , is a construction of order 2 (type: $*_2$) and its type is of order 3, etc. –

Now some analogies linking constructions and concepts can be stated:

constructions	concepts
objective (non-mental)	objective (non-mental)
(in general) complex	(in general) complex
v-construct objects	identify objects

We can see that the only distinction consists in the fact that constructions, in general, contain free variables so that their constructing objects is dependent on some parameters, which we should not expect in the case of concepts. Thus our proposal of explicating *concept* is (where *closed construction* means *construction that does not contain free variables*):

A concept is a closed construction. –

Now we adduce some examples.

Where X is an arbitrary object that is not a construction we will 0X call a *simple concept*. Calling a procedure (construction) simple means that it identifies an object without using any (other) construction. Hence a simple concept is something like ‘immediate identification’ of an object.

Let C be a closed construction that is improper (see above). Then C will be called a *strictly empty concept*. A paradigmatic example: THE GREATEST PRIME. In the area of numbers GREATEST constructs a function G that returns at most one number when applied to a class of numbers (so the type is $(\tau(\sigma\tau))$). PRIME constructs a class P of numbers, so the type is $(\sigma\tau)$. Since it is an infinite class the composition

$$[{}^0G {}^0P]$$

cannot construct any number: G returns nothing when applied to an infinite class.

In contrast to this case such concepts that are general cannot be strictly empty: they at most construct empty classes, but an empty class is an object (unlike ‘empty number’). Thus the concept NUMBERS DIVISIBLE BY 0 is not strictly empty; it could be classified as *quasi-empty*.

Finally, an empirical concept cannot be strictly or quasi- empty: empirical concepts are closed constructions that construct non-trivial intensions, and the latter are non-constant functions from possible worlds; so they are not a kind of empty class. We usually speak about *empirically empty concepts* when the value of the intension that is constructed by the

respective concept in the actual world-time is missing or is an empty class. The first case can be stated when we have concepts like THE PRESENT KING OF FRANCE, the second case, when our concept is, e.g., A GOLDEN MOUNTAIN. So the emptiness of empirical concepts is contingent in contrast to strictly or quasi- empty concepts. (This distinction has been informally stated already by Bolzano.) Notice that if an empirical concept identified the value of the respective intension in the actual world-time rather than the intension alone, then this intuitively accepted distinction between contingent and necessary emptiness could not be formulated either.

Unless an empirical concept is simple it has the form

$$\lambda w \lambda t C,$$

where C is a construction whose free variables are just w (for possible worlds) and t (for time points). To adduce an example of an empirical concept consider

THE NUMBER OF (MAJOR) PLANETS (OF OUR SOLAR SYSTEM)

Type-theoretically: THE NUMBER OF constructs the cardinality N of a class (in our case of a class of individuals), the type of the latter is $(\tau(o_i))$. PLANET constructs a property P of individuals, type $(o_i)_{\tau(o)}$. The whole concept consists in applying the cardinality to the class that is the value of the above property in the respective world-time, and abstracting over worlds-times. So we have

$$\lambda w \lambda t [{}^0N {}^0P_{wt}],$$

where ${}^0P_{wt}$ stands for $[[{}^0P_w]t]$ and $(v-)$ constructs the value of P in the possible world w at the time point t . We can immediately see that this concept does not identify the number 9, it identifies the role a number must play to be the number of planets. So the famous riddle around the incorrect argument

Necessarily, 9 > 7

The number of planets is 9

∴ Necessarily, the number of planets is greater than 7

is easily resolved. The second premiss gets the analysis

$$\lambda w \lambda t [{}^0= [{}^0N {}^0P_{wt}] {}^09]$$

rather than something like

$$[{}^0= {}^0NP {}^09],$$

so that the Leibniz's rule is not applicable: necessity means a quantification over possible worlds whereas the second premiss denotes an empirical proposition. In principle, to apply the Leibniz's rule would mean to disobey a most general logical principle that forbids collision of

variables. Such an elementary mistake cannot, however, be detected unless we explicitly use variables of possible worlds (and time points).

In my view LANL can be construed as a theory of associating expressions with constructions, in particular, with concepts. Many well-known semantic problems and ‘puzzles’ have been already tackled and at least partially resolved using the principles of TIL as suggested above. This concerns propositional and notional attitudes, the distinction *de dicto* vs. *de re*, interrogative sentences, logical analysis of existence, synonymy vs. equivalence etc. Concluding I will adduce a fine-grained analysis of the last-named problem.

Let E, E’ be two expressions, C, C’ concepts (in our sense), O, O’ objects denoted. The line below denotes the boundary separating the realm of logical (conceptual, *a priori*) analysis from the (‘empirical’) reality. We will show a possible analysis of the categories *synonymy*, *(weak) equivalence* and *(contingent) coincidence*.

synonymy		equivalence		coincidence	
E	E’	E	E’	E	E’
	C	C	C’	C	C’
	O	O		O	O’

The value of O, O’
in the actual world-time

This scheme shows that *synonymous expressions* should share the concept, *(only) equivalent expressions* should denote one and the same object *via* distinct concepts, *coincident expressions* should denote distinct objects-intensions that happen to share the value in the actual world.

Thus the least interesting case is surely that of coincidence; notice, however, that just this case played a great historical role in virtue of Frege’s example with morning star and evening star. For Frege these two expressions would be equivalent, for he believed that Venus, the celestial body, is what is denoted by both expressions. Yet we have already suggested that this opinion is untenable and that the genuine object denoted by *morning star*

is an individual role (modelled as the intension of the type ι_{τ_0}) and that the similar role, as denoted by *evening star* is distinct. Venus only happens to fall under both concepts (viz. in the actual world now). The same case of coincidence can be observed whenever we compare two distinct non-equivalent empirical sentences that possess just now the same truth-value. The only object they ‘share’ (by chance) is the truth-value of the denoted proposition in the actual world now.

Summing up, the three relations we have defined in terms of our theory of concepts are:

Synonymy: the same concept

Equivalence: the same intension (empirical case) or the same object
(mathematical
case)

Coincidence: the same value of the denoted intension in the actual world-time.

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