Conceptual Dependence of Verisimilitude  
(Against Miller’s Translation Invariance Demand)  
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The problem of verisimilitude (or more properly: truthlikeness) is, no doubt, one of the fundamental problems in the philosophy of science. As Popper noticed, scientific theories are not true (or rather false) simpliciter but they are in some distance from the truth, the actual state-of-affairs. Thus the ability to determine (to count) exactly the likeness to truth of this or that theory is really important, for otherwise we will not be sure which theory is better than another one or which theory is to be improved in order to be closer to the truth, etc. In his “O jazykovej závislosti niektorých ocenení pravdeblízkosti” (Taliga 2007) Miloš Taliga shows that logicians’ proposals concerning the method of verisimilitude counting are under David Miller’s heavy attack for more than thirty years. Miller repeatedly demonstrated, we are told, that the solution proposed by Pavel Tichý is entirely unsound because the verisimilitude counted by Tichý’s method always reverses, when the respective theory is translated into another language (Miller therefore required translation invariant counting of verisimilitude). Nevertheless, Taliga hides the real truth because he has exposed only Miller’s argument and he has not mentioned any refutation of it. For instance, Tichý himself presented, in his “Verisimilitude Redefined” (Tichý 1976) and “Verisimilitude Revisited” (Tichý 1978), clear arguments distorting Miller’s objections. Thus Taliga told us an incomplete story based only on references to Miller and Popper who chose a downgraded attitude to the predominant way of verisimilitude counting (Tichý, Hilpinen, Niiniluoto, Tuomela, Oddie, and others).

In this paper, I am going to show that Miller’s argument is based on false assumption that verisimilitude is not to be counted with respect to entities verbal formulations of the theories express but with respect to expressions as such, and then mainly that it urges for completely unsatisfiable (and thus absurd) demand that theories based on certain conceptual framework should retain their value of verisimilitude when they are transferred into entirely different framework. Then I shall expose briefly Tichý’s own criticism of Miller. Finally, I shall strengthen its edge by upgrading it to hyperintensional (conceptual) level and explaining thus verisimilitude counting as dependent on conceptual systems.

Theories and likeness to truth
When confronted with external reality, a cognizant subject formulates his findings. The external material world is conceivable as a collection of individuals (universe of discourse) through which they are distributed attributes (properties and relations) in a certain way. A subject testing individuals manages with a set of pre-theoretically given attributes, called by Tichý intensional basis. Modelling attributes as mere classes of individuals (as is usual within extensionalistic construal of language) is inadequate, for a class is given by objects belonging to it. Thus, being a member of a given class is necessary for an individual. Empirical facts, however, are undoubtedly contingent. It is clear that there are plenitude of conceivable distributions – not just a single one – of the same attributes through the same individuals; each of the distributions is called, a bit dramatically, possible world (the collection of possible worlds is called logical space). The actual world is the possible world corresponding (in some way) to the real, material world. Intensional logic then explicates attributes as certain intensions, namely as mappings from possible worlds to classes of individuals (or classes of n-tuples of individuals). Propositions are intensions having truth-values as their functional values. Individuals, truth-values, possible worlds and intensional basis together may be called epistemic framework.

The statements of an investigating subject do not consist only of his particular findings concerned with which specific individual instantiates which particular attribute. Typical subjects (and scientists among them) formulate theories they are going to confirm empirically only. Various scientists frequently formulate different theories dealing with the same individuals and courses of events. We are willing to know then which one of the theories is closer to the truth, i.e., to the actual distribution of attributes through individuals. Tichý thus suggests (and is followed by others) to count verisimilitude as a certain average of the number of wrong guesses about the truth and the number of all particular guesses the theory in question makes. Given the modelling of the content of a scientific theory by means of the propositional calculus, any propositional formula is convertible to its normal form; it can be then minimized and the results are conjunctions combining s.c. constituents, i.e. negated or non-negated atomic formulas as particular guesses. When the truth is, for instance, just h&r&w, then the theory claiming h&¬r&¬w makes three guesses, but two of them being

1 The way how to count verisimilitude was expressed firstly by Tichý in Tichý (1974) and it was accomplished in details firstly in Tichý (1976). (It is worth to note that in his paper “On Explication of the Notion ‘the Content of a Sentence’” from 1966, see Tichý 2004, 55 – 67, Tichý already worked on the semantic content of sentences; moreover, he used the weather example frequently discussed by the theoreticians of verisimilitude.)

2 We will not discuss details or problems of verisimilitude counting. The presented sketchy idea will be sufficient for our purpose.
wrong (2/3). So it is surely less close to the truth than h&r&~w (1/3). Of course, the theory saying h&~r is wrong in one part but it makes only two guesses (1/2), thus it is less wrong than the one saying h&~r&~w, but less close to the truth than the one saying h&r&~w. Tichý (and also others) realized (already in his 1974 paper) that propositional logic is not a sufficient tool for theories modelling because it is more typical for the subject to test individuals as having or not having attributes; thus we need at least first-order predicate logic (capable to handle polyadic predicates standing for polyadic attributes). In order to perform the way of counting sketched above, we have to adopt Hintikka’s distributive normal forms. Notice that, given the proposal just sketched, the verisimilitude of a proposition (denoted by certain sentence) is higher the more possible worlds it sets apart (if we do not take into account the proposition true in no world, the strongest propositions are true just in one world, in world they ‘characterize’).

For the sake of further considerations let us briefly state that explicative framework of Tichý’s transparent intensional logic recognizes also hyperintensional entities called constructions. Construction is a structured (and abstract) way, a procedure, how to arrive at certain object. For instance, there is just one proposition true in all possible worlds but there are (infinitely) many (equivalent) constructions (mathematical theorems, logical tautologies) constructing this unique proposition. Constructions can be viewed as objectual pendants of λ-terms (one-step constructions are pendants of constants; variables, like the possible worlds variable \(w\), are pendants of variable letters; constructions of intensions are of the form \(\lambda w [...w...]\), i.e. they are particular pendants of λ-terms called λ-abstractions, \([FA_t]\) are compositions of certain constructions, they corresponds to λ-terms called applications). Construction expressed by an expression is conceived as its meaning (in a given language), the entity (intension or non-intension) constructed by the construction is conceived as denotatum of the expression. For more details see Tichý (1988; 2004).³

In the last section of this paper, we will utilize certain other notions. Although Tichý repeatedly talked (already from the beginning of the 70th) about basic and derivative attributes (or propositions), he never called them basic or derived concepts. However, Pavel Materna suggested to conceive (certain) Tichý’s constructions as explications of concepts,

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³ We will ignore temporal factor (dependence on time moments) and partiality of functions, which both are implemented within transparent intensional logic.
thus conceptual systems are classes of certain constructions forming their *basic and derivative concepts* (for details see Materna 2004).4

**Miller, languages and theories**

Now Miller’s well-know argument is as follows. Let us consider a language, $L_T$, which has three primitive terms, ‘hot’, ‘rainy’, and ‘windy’; briefly, ‘h’, ‘r’, ‘w’. Imagine then another language, $L_M$, with three primitive terms, ‘hot’, ‘Minnesotan’, and ‘Arizonian’ (‘h’, ‘m’, ‘a’) and suppose that they hold the following equivalences given by objectual definitions:

$$m \equiv_{df} (h \leftrightarrow r)$$
$$a \equiv_{df} (h \leftrightarrow w).$$

Miller shows that when it is the case that it is hot, rainy and windy, the verisimilitude of the theory, $T_1$, namely $\neg h \& \neg r \& \neg w$, is — within the h-r-w language $L_T$ — higher than that of the theory, $T_2$, $\neg h \& \neg r \& \neg w$, because the latter is wrong three times but the former only one time. However, when translated into the h-m-a language $L_M$, the verisimilitude of the theory $T_1$, now $\neg h \& \neg m \& \neg a$, is lower than that of the theory $T_2$, now $\neg h \& m \& a$, because in the present moment it is $T_1$ that is wrong three times, whereas $T_2$ is wrong only one time (we are now comparing both theories with respect to the truth $h \& m \& a$, an equivalent of $h \& \neg r \& \neg w$). The verisimilitude of both theories is thus reversed when we translate them into an equivalent language. Miller concludes that verisimilitude counting suggested by Tichý (and developed also by others) is totally inadequate for it is heavily dependent on languages in which particular theories are formulated. And there is, of course, no reason to prefer one language as superior to another one (Miller 1974, 176; 1976; 1978, 199). Analogous considerations apply to languages using polyadic predicates.5

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4 We will be somewhat inaccurate. For derived concepts (in Materna’s sense) are built up from basic concepts. But conceptual system can be viewed as enriched (it is not Materna’s idea) by *new* concept which is taken as equivalent with some concept already present in the conceptual system. By the term ‘derivative’ we mean to cover either of these possibilities, though we will refer mainly to new concepts introduced by means of some derived concept.

5 It should be said that not only Ilkka Niiniluoto (1978) but mainly Miller (1978) completely ignored that the translation variance argument did apply not only to ‘sentential’ version of Tichý’s verisimilitude counting. For if the argument is applicable at all, Tichý’s proposal of verisimilitude counting on predicates may be the target as well. Niiniluoto happily agreed with Miller’s criticism of Tichý’s ‘sentential’ version (claiming that his own ‘predicate’ version is not touched), but he overlooked Tichý’s ‘predicate’ version. The position of Miller in 1978 is less comprehensible when we realize that in (Miller 1976) Miller criticized Tichý’s ‘sentential’ as well as ‘predicate’ proposal of verisimilitude counting. Remember thus that our ‘sentential’ illustration of the alleged problem on the weather example does not change the fact that analogous (but more
Following Miller, many have accepted that verisimilitude counting suggested by Tichý and others is really a linguistic affair. There is, however, a substantial error in such Milleresque reasoning. As we have seen above, verisimilitude counting is something that is realized on formulas transformed into their normal forms (we will call the process leading to normal forms briefly ‘normalization’). These formulas are conceived as diagrammatic of what theories as linguistic expressions mean. Now it is clear that Miller is fundamentally wrong for two reasons:

a) verisimilitude is counted with respect to normalized formulas and there is no (officially defined) normalization applicable on natural language expressions,

b) verisimilitude is counted with respect to formulas but the formulas are not conceived as expressions belonging to this or that language, these formulas are considered as meanings of theories formulated within various natural languages.

Having Tichý’s system of explication at our disposal, it is clear that the formulas are nothing but Tichý’s constructions. Only such entities are something that is structured; thus counting of distances of constituents is applicable on them. Undoubtedly, one cannot perform normalization on propositions which are mere flat mappings. Verisimilitude counting is to be accomplished with respect to what theories (as linguistic expressions) say; otherwise, there would be dubious English verisimilitudes, French verisimilitudes, etc. It is also absurd to think that formulas in question are purely linguistic entities for there would then be French formulas, English formulas, etc.; this is simply far from truth.\(^6\) It \textit{is Miller’s first error} (but accepted by many) \textit{that verisimilitude is counted with respect to linguistic expressions}. And it is contrary to truth to say with Miller that “If the distance between theories is to depend on their content, rather than on their syntactic form, Tichý’s proposed measure must be deemed unacceptable” (Miller 1978, 199). For it is Miller himself, not Tichý, who carries out counting of verisimilitude on linguistic expressions and not on what these expressions say; Tichý’s account is really semantic, not syntactic. To sum up, formulas like \(~\neg h \& r \& w\) are mere shortcuts for constructions such as \(\lambda w \left[ \neg_0 \text{Hot}_w \land_0 \text{Rainy}_w \land_0 \text{Windy}_w \right]\) that is expressed by the English sentence “It is not hot, it is rainy and it is windy” or by the Czech sentence “Není horko, je deštivo a větrno”. Briefly, \textit{verisimilitude counting is accomplished with respect to}

\(^6\) It would be also fallacious to think, that explication of natural language meanings consists in translations into expressions-formulas of formal language (see Raclavský 2007a).
(normalized forms of) constructions which are expressed by linguistic formulations of theories (this claim will be made more precise below).\textsuperscript{7,8}

There is also another Miller’s awkward error: “Verisimilitude, like truth, will always, if defined, be defined relative to language. But just as truth is language independent… so must judgments of verisimilitude be” (Miller 1974, 176). This incomprehensible couple of sentences shows again Miller’s own fumbling in the heart of the matter.\textsuperscript{9} To unravel the puzzle let us say the obvious: The property being true is fundamentally connected with propositional constructions and thus it is language independent, but the truth of sentence is dependent on the truth of the propositional construction the sentence expresses in certain language; and a propositional construction is true provided the proposition constructed by it is true (i.e. its value at given possible worlds is just T). Analogously, the likeness to the truth is fundamentally the matter of constructions (and subsequently, of propositions constructed by them) and it is language independent; the likeness to truth of sentences, however, is dependent on the verisimilitudes of constructions the sentences express in a given language.\textsuperscript{10}

\textbf{Miller and the invariance of verisimilitude}

\textsuperscript{7} In Tichý (1974) Tichý himself used the term ‘sentences’ in connection with verisimilitude counting and Miller perhaps derived just from this that verisimilitude counting is a linguistic matter. However, he failed to realize that ‘sentence’ is often used by logicians as a term for a formula of certain calculus. Moreover, he failed to realize that formal formulas are offered as presentations of meanings of certain natural language expressions.

\textsuperscript{8} It should be added that Tichý, in his papers on verisimilitude, did not distinguish formulas as expressions from formulas as constructions (btw. the concept of construction is perhaps older than any of his papers on verisimilitude). There is a good reason why not to expose verisimilitude in the middle of 70\textsuperscript{th} in terms of constructions – such proposal would be undoubtedly refuted as idiosyncratic, for already intensional logic was not a paradigm (within the philosophy of science) of these years. Despite of this, what I still miss in Tichý’s papers on verisimilitude is at least short reference to his theory of constructions.

\textsuperscript{9} It should be said that he corrected himself in (Miller 2006, 216) where he offers a similar claim as we will express immediately.

\textsuperscript{10} Of course, it would be absurd to demand verisimilitude of sentences, which would be independent on what the sentences in a given language express (Tichý 1976, 36).

\textsuperscript{11} Although Miller (Miller 2006, 216) has corrected himself with respect to linguistic dependence of truth, he fails to go further and realize that verisimilitude is counted with respect to construction expressed by sentences by means of which theories are verbally formulated. Though I (happily) agree with Miller’s criticism of many theoreticians allegedly avoiding the linguistic invariance problem (cf. chapter 11 of Miller 2006), I cannot understand at all why he still persists in claiming that the mainstream of verisimilitude counting is language dependent. For when syntactic level of language, i.e. expressions, is not taken into account, they remain only constructions. And constructions are extralinguistic objects, therefore, it is absurd to conceive a set of extralinguistic objects as a language and then call verisimilitude counting based on them ‘language dependent’. I am afraid (or rather: aware) that by my main thesis I really touch the neuralgic point of the whole language dependence controversy.
Miller’s unqualified shift from language semantic content (i.e. from constructions) of theories to the syntactical level is only a minor error in his inquiry of verisimilitude counting. Had it been exposed alone, theoreticians might quickly disclose its fraudulent character. However, the peril of the shift is in that it covers a much deeper fallacy in Miller’s argumentation. To illustrate the fallacy, let us suppose that two subjects, S₁ and S₂, standing side by side, measure the distance of an object O occurring directly before them. Suppose further that O is equidistant from S₁ and S₂ and that the distance is actually one meter. But S₁ uses a system of measurement having one meter as its key length measure, whereas S₂’s system of measurement has one yard in its stead. S₁ and S₂ suggest their guesses, i.e., theories Tₛ₁ and Tₛ₂ concerning the distance of O from them. Now before their comparison, one naturally expects that it is necessary to reformulate Tₛ₁ and Tₛ₂ into one and the same system of measurement first. Only after that it is reasonable to count which of Tₛ₁ and Tₛ₂ is closer to the truth. Surprisingly, Miller ignores to state such natural requirement but this is not the worst thing he does. Contrary to expectation of anybody, Miller postulates a demand to the effect that – independently of systems of measurement on which any couples of theories Tₛ₁ and Tₛ₂ are based – when the theories are, for example, equally right (they express just the truth), then they should state the distance from O with the help of the exactly same number. For example, when Tₛ₁ says the truth, it must state that the distance is just 1 (in meters) and Tₛ₂ also saying nothing else but the truth must state that the distance is just 1 (in yards). Exactly similar ‘logic’ underlies Miller’s way of reasoning – “There can... be no logical reason for... tolerating variations in distance as we move from one to other.” (Miller 1976, 199; analogously in his other papers).

In the weather example the role of systems of measurement is played by intensional bases IBₜ={h,r,w} and IBₘ={h,m,a}. Instead of equivalence 1 yard \( \equiv df \) 0.9144 meter, we manage equivalences m \( \equiv df \) (h\( \leftrightarrow \)r) and a \( \equiv df \) (h\( \leftrightarrow \)w). So it holds that h&m&a (unpacked as h&(h\( \leftrightarrow \)r)&(h\( \leftrightarrow \)w)) is equivalent to h&r&w, ~h&~m&~a is equivalent to ~h&r&w, ~h&m&a is equivalent to ~h&~r&~w, and so on. Now suppose that the truth is just that it is hot, that it rains, and that it is windy, i.e. h&r&w (or h&m&a). But S₁’s theory Tₜ (formulated upon IBₜ) says ~h&~r&~w, so its verisimilitude is rather low. However, S₂’s theory Tₘ, formulated upon IBₘ, says ~h&~m&~a. If we prefer the framework of IBₜ as that one where the verisimilitude of both theories should be determined, then we must transfer Tₘ into that framework. So the content of Tₘ is then ~h&r&w and its likeness to truth (1/3, i.e. 0.33) is

\[\text{For its 3 wrong guesses the distance is 3/3, i.e. 1; the distance of h&r&w is 0/3, i.e. 0, what is the best distance from the truth, the entire closeness to it.}\]
higher than that of \( \sim h \& \sim r \& \sim w \). However, this is not what Miller exactly wanted to hear: he let us enjoy the transfer of \( T_T \) in the framework upon \( IB_M \), so we get \( \sim h \& r \& w \) with the verisimilitude 0.33 from the truth \( h \& m \& a \), whereas the verisimilitude of \( \sim h \& \sim m \& \sim a \) is 1. Notice that Miller does not waste our time by drawing an attention to the fact that he has changed the measure system (intensional basis), which is very important factor here. He rushes rather to conclude that the truth is the same as before (it is captured jointly by \( h \& r \& w \) and \( h \& m \& a \)), but the distances of \( T_T \) and \( T_M \) from the truth are now suspiciously reversed. As we have cited above, the only verisimilitude counting Miller is going to accept must not change the value of verisimilitude, i.e. must be the same, even if we move (completing here Miller’s dictum) from the framework based on \( IB_T \) to the one on \( IB_M \).

In the example with measure systems based on meters or on yards it is plain that Miller’s key demand is not reasonable at all. The reason is clear – it is completely unsatisfiable.\(^{13}\) The absurdity of Miller’s demand is also plain given its two unacceptable direct consequences. If we would be ignorant (as Miller) of the differences of intensional bases, we might demonstrate by his method a futile claim that both \( \sim h \& r \& w \) and \( \sim h \& m \& a \) are theories equally distant from the truth,\(^ {14}\) but together they are inconsistent (for \( \sim h \& m \& a \) is \( \sim h \& \sim r \& \sim w \), so we get \( \sim h \& r \& w \) and \( \sim h \& \sim r \& \sim w \), which is equal to contradiction). Of course, the joint inconsistency holds for any such couple of theories with ‘the same’ verisimilitude (except one couple, namely \( h \& r \& w \) and \( h \& m \& a \)).\(^ {15}\) Another absurd consequence which should gobble the admirer of Miller’s ignorance of measure systems (intensional bases) relativity is that by the suitable equivalences we can demonstrate, by Miller’s method, that each theory (which does not hit the truth) is equally furthermore from the truth as any other theory. An example supporting such reasoning is already made for quite good theory \( \sim h \& r \& w \), because the equivalences as regards to \( m \) and \( a \) can be used to show that \( \sim h \& r \& w \) is convertible to \( \sim h \& \sim m \& \sim a \), which has the greatest distance from the truth (\( h \& m \& a \)).

Tichý against Miller

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\(^{13}\) Of course, unless the measurement systems in question are not one and the same.

\(^{14}\) The verisimilitude of \( \sim h \& r \& w \), given \( IB_T \), is 0.33, the verisimilitude of \( \sim h \& m \& a \), given \( IB_M \), is 0.33. But a reader may perhaps reverse the bases and prefer rather the concurrent verisimilitude of those theories equal just to 1.

\(^{15}\) Now we can also rush to the hasty conclusion that the only consistent possible world is the actual one and that other possible worlds are only inconsistent fictions on the part of logicians.
As a matter of fact, Tichý (as well as others) does not upbraid Miller for the shift from theories contents to their verbal, syntactical, representations. Nevertheless, in Tichý (1976) as well as in Tichý (1978) Tichý rejected that couples of sentences like ‘h&r&w’ and ‘h&m&a’ are mutually intertranslatable, an assumption on which Miller insisted (e.g., Miller 1978, 199). The argumentation is as follows (Tichý 1978, 192). (a) Two intertranslatable expressions are always co-denotative; (b) sentences denote propositions; (c) propositions are function from possible worlds to truth-values.\(^\text{16}\) The principles (a) – (c) together imply that sentence V\(_1\) of L\(_T\) that is translatable into sentence V\(_2\) of L\(_M\) should be such that both V\(_1\) and V\(_2\) denote the very same proposition. But propositions denoted by sentences of L\(_T\) are functions from possible worlds over IB\(_T\), whereas propositions denoted by sentences of L\(_M\) are functions from possible worlds over IB\(_M\). However, the possible worlds forming the logical space LS\(_T\) (over IB\(_T\)) and those forming the logical space LS\(_M\) (over IB\(_M\)) are entirely distinct because they consist of assignments of truth-values (extensions) to primitive medadic (i.e. 0-adic) attributes from the respective intensional bases which are different. Hence members of LS\(_T\) are ‘bold’ entities like \{hot→F, rainy→T, windy→T\}, whereas members of LS\(_M\) are entities like \{hot→F, Minnesotan→T, Arizonan→T\}. Thus no proposition over LS\(_T\) is identical with any proposition over LS\(_M\). (Tichý 1976, 35; 1978, 193.) Therefore, if two languages are not based on the same epistemic framework, no sentence of the former is intertranslatable with any sentence of the latter (Tichý 1978, 193). Consequently, if a theory formulated within L\(_T\) is not translatable into the theory formulated within L\(_M\), one cannot object to Tichý’s theory assigning to them distinct verisimilitudes (Tichý 1978, 193).\(^\text{17,18}\)

The simple and natural fact that intertranslatable languages have to share one and the same epistemic framework is not be confused with the entirely distinct fact that the conceptual system of one epistemic framework can be enriched by introducing new,

\(^{16}\) Tichý adds, in footnotes (1978, 195 – 196), that from verbal discussions with Miller and his followers it followed that they wish to refute Leibniz’s identity of indiscernibles principle or that intertranslatable sentences denote the same proposition or the principle of individuation of propositions. Fatal consequences of the refutation of any of these principles are discussed by Oddie (1981, 251 – 252).

\(^{17}\) If two languages are not intertranslatable, no definition in meta-language can introduce into one of the languages expressions intertranslatable with expressions of the second language. No definition (Miller seems to be hoping to apply) may ignore what expressions of the languages say.

\(^{18}\) Barnes (1991, 323 – 324) lacks the notion of the propositional construction, nevertheless he reached similar conclusion as Tichý (or me). For due to Barnes, knowledge-beliefs formulated by means of hrw-language and hma-language are different despite their equivalence. Put as it is, it is suspicious, of course. But in our terms we can paraphrase Barnes’ claim as follows: a) both ‘~h&r&w’ and ‘~h&m&a’ denote the same proposition, thus they are equivalent, but the ‘contents’ of both sentences are epistemologically different beliefs (meanings of these sentences are different propositional constructions), b) hrw-language and hma-language are ‘epistemically independent’ for they express different constructions (or, as Tichý already showed, they are based on different intensional bases, they are based on entirely different epistemic frameworks).
derivative, attributes by means of ‘meaning postulates’.¹⁹ For example, we may define, over IBₜ, new attributes based on the original basic ones by means of (objectual) definitions like \( m \equiv_{df} (h \leftrightarrow r) \), \( a \equiv_{df} (h \leftrightarrow w) \) (or alternatively by \( m \equiv_{df} ((h \& r) \lor (\sim h \& \sim r)) \), \( a \equiv_{df} ((h \& w) \lor (\sim h \& \sim w)) \)).²⁰ As has been already pointed out by Tichý, all equivalence statements and intertranslatability statements made by Miller are true in such extension of Lₜ. But one can then refer to the obtaining state-of-affairs that it is hot, rainy, and windy by the sentence expressing \( h \& r \& w \) or alternatively by the sentence expressing \( h \& m \& a \). But as soon as \( \sim h \& \sim m \& \sim a \) is in fact \( \sim h \& r \& w \), its verisimilitude is surely the same as that of \( \sim h \& r \& w \), namely 0.33 (analogously for other alternative forms of conjunctions of constituents \( h, r, w \)). (Tichý 1978, 194.) Tichý did not say here explicitly – but he in fact showed it – that verisimilitude must be counted with respect to the basic attributes of given intensional basis, which give rise to the respective combinatorics of possible worlds. As we will see also later, it is an insufficiency of Tichý’s approach that he did not expose explicitly the dependence on conceptual systems. (Next section is mainly aimed at repairing this drawback.)²¹ ²²

Facts, possible worlds and conceptual systems

Some theoreticians would not accept Tichý’s argumentation as it is given by him, some would not understand (or grasp) its certain key features.²³ One may object, for instance, that

¹⁹ This hidden equivocation on which Miller’s argument is based was observed already by Tichý (1978, 193 – 194).

²⁰ Of course, we can introduce into IBₘₐ derivative attributes rainy and windy by means of \( r \equiv_{df} (h \leftrightarrow m) \) and \( w \equiv_{df} (h \leftrightarrow a) \).

²¹ On the other hand, Niiniluoto explicitly said – and this is entirely correct view – that “degrees of truthlikeness should be relativized to conceptual systems” (Niiniluoto 1978, 310). In this particular case, however, Niiniluoto did not show, unlike Tichý, how the verisimilarities counting should look like. It seems to me, but I cannot go here in details, that Niiniluoto did not realize clearly what conceptual systems are, how they are related to sentences or verisimilitude (for example, he accepts that two languages are intertranslatable despite the fact that they have different conceptual systems at their disposal; below we will see that this is impossible; or: in Niiniluoto 1987, 451, he argued that Miller’s argument did not arise for we count verisimilitude with respect to atomic propositions – the view serving as the reason is right, as we will also show below, but then verisimilitude is ‘atomic propositions’-relative and this alone cannot discredit Miller’s argument).

²² In Britton (2004) we meet (but I think not really persuasive) way how to avoid Miller’s argument. It is based on the assumption that we count verisimilitude only with respect to partial (not ‘whole’) properties. In fact, I cannot understand this reasoning for properties are simply properties, not their own fragments; handling with fragments has hardly anything to do with degree of theories’ truthlikeness.

²³ Miller was the first one. For he curiously concluded that Tichý’s claims about not intertranslatability of languages based on distinct epistemic frameworks means that the only intertranslatable languages are those with the same atomic sentences (Miller 1976, 364), what amounts to say, for instance, that English is not intertranslatable with French, as millions people believe, but only with itself. From this it is apparent that Miller was unable to recognize other entities than sentences, that he was not able to understand propositions as entities associated with sentences. No surprise that he conceived his alleged refutation of verisimilitude
world described by ‘h&r&w’ is exactly the same as that described by ‘h&m&a’ (analogously for ‘~h&r&w’ and ‘~h&~m&~a’, etc.). To draw the conceptual distinction where it is needed one, let us illustrate here the logical space underlying LT, i.e. LS_T (adopted from Tichý 1978):

\[
\begin{array}{cccccccc}
 & A & B_1 & B_2 & B_3 & C_1 & C_2 & C_3 & D \\
\hline
\text{hot} & T & T & T & F & T & F & F & F \\
\text{rainy} & T & T & F & T & F & T & F & F \\
\text{windy} & T & F & T & T & F & F & T & F \\
\end{array}
\]

and LS_M underlying L_M.\(^2^{24}\)

\[
\begin{array}{cccccccc}
 & E & F_1 & F_2 & F_3 & G_1 & G_2 & G_3 & H \\
\hline
\text{hot} & T & T & T & F & T & F & F & F \\
\text{Minnesotan} & T & T & F & T & F & T & F & F \\
\text{Arizonan} & T & F & T & T & F & F & T & F \\
\end{array}
\]

Tichý claims that A and E (similarly, H and B_3, etc.) are not one and the same world because A has as its ‘reverse’ the world D, whereas E has as its ‘reverse’ the world H (Tichý 1978, 193). However, one cannot easily believe this because when A or E happens to materialize, molecules of air move quickly, water drops fall down from the heaven, portions of air clash with themselves. Similarly, when we think what happens when it holds ‘~h&~m&~a’, then molecules of air do not move quickly, but water drops fall down from the heaven and portions of air clash with themselves, what is described also by ‘~h&r&w’. Thus we get the following identities of worlds:\(^2^{25}\)

\[E = A, \quad F_1 = B_1, \quad F_2 = B_2, \quad F_3 = D, \quad G_1 = C_1, \quad G_2 = C_2, \quad G_3 = C_3, \quad H = B_3\]

It should be appreciated, however, that what is materially realized is not a possible world, but just the real world, the entity which simply ‘is’. However, possible worlds are something what ‘can hold’, what can be the right conceptual grasping of the real, material world. Possible worlds are not collections of material courses of events, material things. Possible worlds are conceivable distributions of attributes through certain items, they are collections of (mutually non-contradictory) possible facts. Possible worlds are thus immaterial, conceptual entities. Moreover, we already explicated the material world as a collection of individuals, not as a collection of facts.

\(^{24}\) In each column of each table no proposition is diagramed, these tables illustrate only two logical spaces (i.e. possible worlds).

\(^{25}\) To know what happens if it holds ‘~h&~m&~a’ check the semantic truth-table for ¬h&(¬(h↔r)&¬(h↔r)), you will find that it is true only when h gets F, but both r and w get T. Analogously for other such formulas.
But what is a fact? One of our deepest pre-theoretical convictions as regards the notion of fact (or state-of affairs, if you wish) is that Xenia’s being a woman is a fact consisting in Xenia’s instantiating the property being a woman. Hence, fact is a structured entity. It is a thought (as Frege claimed), it is a proposition (but not in the modern sense) which is composed from the component concepts. It is a thought that may be true; the conceivable truth of fact is our second fundamental conviction concerning fact. Within the explicative framework of Tichý’s transparent intensional logic we have two reasonable possibilities how to explain the pre-theoretical notion of fact. If we choose, as Tichý did (2004, 1988), propositions as such entities, our pre-theoretical intuition conceiving facts as structured entities is inevitably thrown away: for propositions are structureless mappings associating truth-values with possible worlds; there is no track of any individual or attribute in them at all. Therefore, I prefer (Raclavský 2007) hyperintensional explication of facts, according to which facts are explicated similarly as (Frege’s) thoughts, i.e. as propositional constructions. So they are structured and they are composed from constructions which are explications of concepts. (That Xenia is a woman, for example, consists then in combining the single concept of Xenia and the single concept of the property being a woman.) Being such, facts are related to conceptual systems, so we can distinguish basic facts and derivative facts.²⁶

Before going further let us expose another, perhaps more convincing, objection to Tichý’s refutation of the intertranslability in question. The objects from our explicative basis (like particular truth-values, individuals or possible worlds) should be primitive, not ‘bold’ entities. It would be a moving circle to explain, for instance, propositions as functions from possible worlds and then to explicate possible worlds as collections of certain propositions. One of these entities has to be taken as primitive, non-analyzable and there are good reasons why possible worlds should be chosen as primitive. Being unanalyzable means that no internal content, or ‘structure’, of such entity is taken into account within our explicative system. Thus, possible worlds are simple entities conceived only as numerically distinct from each other. (These claims were adopted from Tichý 1988, §36.)²⁷ Being such, possible worlds can easily be named by proper names (we will designate such possible worlds by non-italicized capital letters). Thus A and E, for example, are indiscernible for no internal ‘structure’ is reflected (‘A’ and ‘E’ are only distinct names of the same world), they both represent the same state of weather (see the above identities and the following paragraph).

²⁶ This enables us to shake off the dual two-sided aRb/bRa fact, which led Tichý to the adoption of propositional explication of facts.
²⁷ Possible worlds taken as unanalyzable are more preferable than Tichý’s original ‘bold’ ones.
Now the respective logical space underlying both languages, $L_T$ and $L_M$, contains just 8 worlds and $2^8$ propositions. As a result, the sentence ‘$\neg h \& \neg m \& \neg a$’ seems unproblematically intertranslatable with the sentence ‘$\neg h \& r \& w$’ for they may denote one and the same proposition. This is perhaps a tacit objection of theoreticians against Tichý’s claim that these two sentences (theories, languages) are not mutually intertranslatable.

However, the judgment about intertranslatability is not conclusive either. Seeing that there are many quite different sentences (of one language) which denote one and the same proposition, the meanings of these sentences must be individuated in a more fine-grained way than by propositions (i.e. sets of possible worlds); meanings of sentences should be rather construed as propositional constructions. Then we should strengthen the condition of suitable translation to $(a^*)$: Sentence $V_1$ of one language is translatable into sentence $V_2$ of another language only if both sentences express the very same propositional construction. For instance, ‘$\neg h \& \neg m \& \neg a$’ is not intertranslatable with ‘$\neg h \& r \& w$’, for the first sentence expresses the construction (written in a truncated way as) $\neg h \& \neg m \& \neg a$, whereas the second expresses the construction $\neg h \& r \& w$. I.e., we do not accept Miller’s intertranslatability assumption. Clearly, if two sentences (or theories) are really intertranslatable, they express the same construction; the verisimilitude of this construction is, of course, immune to translation.

In order to forestall an objection that $A$ and $E$ are in fact one and the same possible world in a better way than it did Tichý, we turn our attention to our conceptual grasping of phenomena which materialize in the real world. We have claimed that within our system of explication possible worlds are primitive, unanalyzable items. But no system of explication (i.e., a system of functions) explicates anything until it is clear which entities are represented by the primitives, i.e., until it is understood, for example, that T and F represent affirmative and negative quality respectively. (Tichý 1988, §38.) Analogously for possible worlds: mere A does not explicate anything until it is clear which pre-theoretically conceived distribution of attributes through individuals it represents. A represents, one is tempted to say – and rightly –, Tichý’s ‘bold’ possible world, distribution $A$. Now my question is as follows: Are possible worlds as distributions further explicable? Do not we conceptually grasp them? I am strongly convinced that yes. Therefore, I suggest (see also Raclavský 2007) to explicate possible worlds in the hyperintensional way as certain collections of (mutually non-contradictory) facts, i.e. as collections of propositional constructions (we will designate such construed possible worlds $A^*$, $B_3^*$, etc.). This adheres to our view that possible worlds are particular conceptual grasps of the real world, that possible worlds are certain conceptual entities. Of course, any possible world explicated in this way is relative to a conceptual system. Thus we
have, for instance, conceptual system CS\textsubscript{T} which has among its basic concepts constructions 0\text{Hot}, 0\text{Rainy}, 0\text{Windy} (and 0\neg, 0\&\text{, etc.}) and among its derivative concepts constructions 0\text{Minnesotan} and 0\text{Arizonan} (given by the objectual definitions 0\text{Minnesotan} \equiv_{\text{df}} [0\text{Hot} 0\leftrightarrow 0\text{Rainy}] and 0\text{Arizonan} \equiv_{\text{df}} [0\text{Hot} 0\leftrightarrow 0\text{Windy}]).\textsuperscript{28} Then A* is a possible world consisting of basic (propositional) concepts of CS\textsubscript{T}, namely 0\text{Hot}, 0\text{Rainy}, 0\text{Windy} (this possible world may be described by the sentence that expresses the ‘characteristic’ of this world, namely the construction \lambda_{w} [0\text{Hot}_{w} 0\& 0\text{Rainy}_{w} 0\& 0\text{Windy}_{w}]). Needless to say that A* is apparently distinct from E*.\textsuperscript{29}

It is evident that we are returning to various thoughts already given above; we only give them hyperintensional (conceptual) interpretation. The verisimilitude of the theory saying \lambda_{w} [0\neg 0\text{Hot}_{w} 0\& 0\text{Rainy}_{w} 0\& 0\text{Windy}_{w}] is counted as the average of wrong guesses with respect to A* (if A* is considered as the possible world which is actual) and the number of guessess-constituents (i.e. 0\neg 0\text{Hot}_{w}, 0\text{Rainy}_{w}, 0\text{Windy}_{w}). However, before we count the verisimilitude of the theory saying, for instance, \lambda_{w} [0\neg 0\text{Hot}_{w} 0\& 0\text{Minnesotan}_{w} 0\& 0\neg 0\text{Arizonan}_{w}] we have to convert this construction to \lambda_{w} [0\neg 0\text{Hot}_{w} 0\& 0\text{Rainy}_{w} 0\& 0\text{Windy}_{w}] by, firstly, replacing derivative concepts by basic ones and by, secondly, equivalences (and normalizations). Hence: the verisimilitude is counted with respect to what the theory says, i.e., propositional construction, which has to be transformed in order to contain only basic concepts (and is turned into its distributive normal form) of a conceptual system and then with respect to the possible world we considered as actual and which is explicated as the collection of basic facts, i.e. propositional constructions composed from basic concepts of the very same conceptual system. Notice that conversion to basic concepts is really inevitable: if we have 0\text{Minnesotan} as a derivative (propositional) concept, we cannot count whether it is closer to the truth A* than \lambda_{w} [0\neg 0\text{Minnesotan}_{w}]. The reconciliation of derivative concepts with respect to the basic ones is necessary; 0\text{Minnesotan} has to be converted to the (equivalent)

\textsuperscript{28} None of these definitions must be closed by [\forall [\lambda_{w} (being not closed, the definition should be satisfied by any valuation for w)].

\textsuperscript{29} My proposal can be viewed as a suitable hyperintensional (constructional) explanation of Graham Oddie’s proposal suggesting to conceive possible worlds as structures and theories as structures somehow distant from the structure of the actual possible world (see Oddie 1981, 252, it was accepted by Volpe 1995; Oddie 1986, 152). In fact, the original Oddie’s proposal with ‘permutation of individuals’ cannot be easily understood, moreover, it can be easily misinterpreted (as shown by, for instance, Uhrbach 1983). Among other theoreticians that do not accept Tichý’s or Oddie’s defence and accept then Miller, there are Pearce (1983), Barnes (1995) (due to my arguments given above, they are obliged to share Miller’s fallacious construal of language and theories; for the case of Barnes, notice that he had not really clear conception of language in 1991, cf. above). Bonilla (1992) accepts structures and is, thus, immune to Miller’s argument; but he prefers set-theoretical construal of structures thus his proposal is not as clear as ours (Tichý’s constructions are not set-theoretical, they are ‘algorithmic’).
construction containing only basic concepts from which the possible world the distance from which we are going to count is composed.30

Of course, if possible worlds are based on distinct conceptual system, say CS_M, the contents (i.e. constructions) of all theories we wish to compare with respect to their likeness to truth have to be converted in order to be composed only from the basic concepts of CS_M. Notice, that we count the distance from a possible world, i.e. a genuine structure, not from the real, material world. Because the real world must be conceptually grasped first. Only then it is possible to measure the distance of theories from it. But our conceptual grasps of the real world are, as a rule, relative to conceptual systems. Consequently, the verisimilitude is inevitably relative to conceptual systems.31 Conceptual systems are entities similar to systems of measurement we thought about in our example concerning measurement of the distances of O from S_1 and S_2. When we move from one system of measurement to another, values of lengths should (or rather: have to) change. It would be absurd to require, as Miller in fact did, that they be ‘absolute’, thus unchanging. Since there is no privileged, ‘absolute’ conceptual system, values of verisimilitudes (of various theories) have to be allowed to change, being conceptual systems relative.32 Thus within CS_T, the distance of the theory T_T expressing \( \lambda w \[0 \sim Hot_w \& 0 \sim Rainy_w \& 0 \sim Windy_w \] \) from A* is 1 and the distance of the theory T_M expressing \( \lambda w \[0 \sim Hot_w \& 0 \sim Minnesotan_w \& 0 \sim Arizonan_w \] \), converted to \( \lambda w \[0 \sim Hot_w \& 0 \sim Rainy_w \& 0 \sim Windy_w \] \), from A* is 0.33. But when we change the conceptual system – and thus also the possible world with respect to which we measure the distances (this is very important) – to CS_M, then it is natural as well as lawful that the verisimilitude of the theory T_T

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30 It should be specified that verisimilitudes counting is related to the possible world we consider as actual (obtaining). But generally we may count the distances from any possible world, so verisimilitude can be viewed only as a special case of distance (analogous proposal was already made by Raimo Tuomela in Tuomela 1978). Notice also that we can count distances of possible worlds from the one we considered as actual; this was proposed (most probably not within hyperintensional but only within intensional framework) by Risto Hilpinen in 1976 (the paper was not accessible to me).

31 Niiniluoto’s treatment of this problem in the sense of ‘pragmatic ambiguity’ (already in 1977, but I can refer only to Niiniluoto 1984, 18) is rather misleading. Our choice of certain conceptual system (within certain historical situation) is undoubtedly a pragmatic matter. On the other hand, all concepts as well as conceptual systems are given. This implies that relativization of possible worlds and theories to conceptual systems – and then of their verisimilitude – is strictly objective. The choice of conceptual system with respect to which we count verisimilitude is, of course, a pragmatic matter; this does not mean, however, that counted values of verisimilitude are not objective. It is entirely unnecessary to give up the ‘objective, regulative idea’ (early Popper’s view), despite the counting we are in fact able to provide (when occurring in certain historical situation). (I completely agree with Niiniluoto’s criticism, ibid., of Uhrbach 1983, that for certain pragmatics involved, verisimilitude is insignificant and uninteresting; in my view, Uhrbach mistakes more than it seemed to Niiniluoto.)

32 One can find certain convergent ideas in (Niiniluoto 1982, 441) where he criticizes Sellars’ view on the basis that there are more conceptual systems correctly conceptualizing the world (in the sense ‘real world’). Niiniluoto also realizes that the real world is not directly pictured, that languages picture rather conceptual grasps of the real world whereas these ‘conceptual worlds’ consist of facts.
expressing $\lambda w [0 \sim 0 \text{Hot}_w \ 0 \& 0 \sim 0 \text{Rainy}_w \ 0 \& 0 \sim 0 \text{Windy}_w]$, converted to $\lambda w [0 \sim 0 \text{Hot}_w \ 0 \& 0 \sim 0 \text{Minnesotan}_w \ 0 \& 0 \sim 0 \text{Arizonan}_w]$, from $E^*$ (not $A^*$) be now 0.33 and the distance of the theory $T_M$ expressing $\lambda w [0 \sim 0 \text{Hot}_w \ 0 \& 0 \sim 0 \text{Minnesotan}_w \ 0 \& 0 \sim 0 \text{Arizonan}_w]$ from $E^*$ be 0.33. Verisimilitudes are reversed similarly as lengths of $O$ from $S_1$ and $S_2$ (stated by $T_{S_1}$ or by $T_{S_2}$) are changed when we change the system of measurement.

Notice that reversions of verisimilitudes depending on conceptual systems do not imply that verisimilitude counting is useless from the very beginning. If we wish to know exactly how close to the truth two theories are, we must, firstly, relativize the truth to some conceptual system and, secondly, transform theories in order to be based on the same conceptual system. But once these two (preliminary) matters are fixed, mathematically exact measurement of the two theories propinquity to the truth was given. And we want to know, regarding the theories, just numerically precise degrees of their likeness to the truth. Completely analogous results of verisimilitude counting are expected, if the truth is based on a different conceptual system. The task here is to find mathematically exact degree, not the only ‘absolute’, cogent number. In order to require just one cogent values of verisimilitude(s) one must firstly choose a conceptual system (and then the truth) which should be the cogent one. It is apparent, that Miller (completely ignorant of the changes of conceptual systems and dependences of verisimilitudes on them) in fact appeals to us to consider such preferable, fundamental conceptual system. One must not confuse, however, this desirable wish for unique, cogent conceptual system with the method how to count verisimilitude exactly – one must not complain that such mathematical method does not decide on which conceptual system the truth should be based. The task of a suitable counting method proposal was to find the exact truthlikeness counting. Only provided a cogent conceptual system (and the truth) is fixed one can justifiably require – and if the proposed counting of verisimilitude is correct, he will really get – the cogent value of theories’ verisimilitude.33

References


33 The author is actually supported by the GAČR grant no. 401/07/P280.


preprint URL:

34 All Tichý’s papers are reprinted in (Tichý 2004).