Tichý’s Possible Worlds

Abstract: Pavel Tichý originally published his conception of possible worlds in 1968. Even though he modified it over the following twenty-five years, its core remained unchanged. None of his thirty journal papers or books containing the notion of possible worlds was a study in metaphysics, Tichý always introduced the notion in the context of other investigations where he applied his Transparent intensional logic either to the analysis of natural language or explications of other notions. The unique character of Tichý’s conception makes it difficult to classify; but to understand it as combinatorialism combined with actualism seems to be admissible. When explaining and commenting on Tichý’s conception, I also suggest its possible alternative development.

Keywords: possible worlds; combinatorialism; actualism; possible world semantics; Transparent intensional logic.

1. Introduction

It is well known that the modern story of possible worlds began with Wittgenstein’s states-of-affairs (1921/1922). Carnap (1947) introduced a linguistic mirror of this conception (viz. state-descriptions) to formal semantics. Through a development best summarized elsewhere (e.g. Copeland 2006), Kripke (1963) and others supplemented formulas of modal logic with model-theoretic semantics based on the notion of possible worlds. During the 1970s, Lewis proposed basics of his theory (1973), Plantinga (e.g. 1970), Adams (1974), Stalnaker (1976) and others significantly contributed to the debate. As the reader will be aware, the contemporary debate revolves around Lewis’ landmark monograph (1986); further notable books include Armstrong (1989), Divers (2002), Nolan (2002) and Yagisawa (2010).

The present paper deals with a remarkable conception of possible worlds developed by the post-Prague Spring refugee Pavel Tichý (1936 Brno – 1994 Dunedin) who moved to New Zealand where he became a professor of philosophy. Tichý never wrote a systematic paper on possible worlds, nor a book, which is one of the reasons why his conception is not well known. Another reason might be simply that he exposed the notion outside metaphysical debate, in the context of different investigations – where he applied his system of intensional logic.
My main objective is to present Tichý’s proposal, while focusing on its recent version. Tichý developed his views in four stages:

1) around 1969 (the papers published in 1968, 1969 and 1971)
2) 1971–79 (especially the unpublished book 1976 and a number of related papers mainly in the late 1970s)
3) between 1980 and 1988 (partly in cooperation with G. Oddie)
4) 1988 and after (his supreme proposal published mainly in 1988).

However, such division is somehow artificial because the stages are overlapping and a number of the main ideas persisted throughout.

None of Tichý’s expositions of the notion are extensive: they are usually one page or less. Tichý’s most comprehensive treatise can be found in his excellent monograph “The Foundations of Frege’s Logic” (1988, hereafter FFL), but the core part of its section “36. Possible Worlds” covers only three pages. Obviously, I have had to fill at least some gaps in order to present the theory. I will also add explanatory remarks that should help the reader to compare Tichý’s conception with other proposals.

These limitations as well as the limits of the paper format, force me to omit a deeper discussion or defence of Tichý’s position. It should also be kept in mind that the contemporary understanding of possible worlds and the classification of their conceptions was proposed later (cf., e.g., Haack 1978, Menzel 2014) than Tichý developed his views. Since Tichý is a distinctive thinker, his conception is sometimes difficult to subsume into one particular category.

In the following Section (2), I explain Tichý’s first conception followed by an immediate assessment (Section 3). In Section 4, I introduce its second and third stage. Then, in Section 5, I explain Tichý’s supreme conception. In Section 6, I briefly explain his intensional logic and semantics and provide a conclusion (Section 7).

2. The first stage: procedures and intensional basis

During the late 1960s, Tichý faced limits of analysis of our conceptual scheme by means of classical extensional logic. Being well acknowledged with algorithms and related problems, he proposed in his textbook introducing logic as a framework of science (1968a), to enrich
predicate logic by means of \( \lambda \)-calculus developed by Church (e.g. 1940). From a debate between Tichý (e.g. 1966) and Czech extensional logicians it follows that he knew Tractarian/Carnapian ideas concerning the content of empirical/non-empirical sentences, whereas the latter ones are valid in all circumstances.


As Tichý explained (1969, 7–9), the aim of empirical investigation is to provide results of tests or procedures examining which attributes (i.e. properties and relations) are possessed by investigated objects. The sentence such as “The object \( X \) is heavier than \( Y \)” is a record of this: it says that \( X \) was positively tested on having that attribute.

To verify the sentence, one executes the procedure consisting of testing \( X \) on being such and such. The meaning of the sentence is identified with a complex concept – the procedure. Procedure is an older term for algorithm. Procedures split into two kinds, empirical and non-empirical. For an execution of any empirical procedure, one has to introduce state of the external world.

According to Tichý (1969, 9), language is based on an empirical system which consists of a particular fixed finite set of individuals and a fixed register of elementary tests, such as \( \text{BE HEAVIER THAN} \ Y \), called intensional basis.

Tichý always repeated this idea, only deleting “finite” and replacing “empirical system” with “epistemic framework” and “set of individuals” with “universe of discourse”. He admitted partial attributes to be in intensional basis (cf., e.g., Oddie, Tichý, 1982, 234, 3.1.a). Tichý also dismissed the idea of intensional independence of tests employed in (1969) because he soon realized the force of Kemeny’s (1951) objection to Carnap (and Wittgenstein) according to which some intuitive attributes depend on others; in other words that atomic facts are not necessarily independent.

Possible worlds are systems of possible outcomes of applications of procedures collected in intensional basis:

each combinatorial possibility as to the outcome of applications of all the tests in the intensional basis to all individuals (or to couples of individuals etc.) must be regarded as a conceivable state of the external world. Let us call these possibilities briefly possible worlds with respect to the empirical system (Tichý 1969, 9)
Tichý also counted the number of possible worlds, which reminds us of Wittgenstein’s counting in (1921/1922, 4.42). Tichý added that pure semantics cannot decide or assume which world is the actual one.

The rest of Tichý’s paper focuses on defining *intensions* as classes of equivalent procedures. Intensions were thus not functions from possible worlds (Ws) as we know it from possible world semantics and Tichý’s later writings.¹

In the subsequent paper “An Approach to Intensional Analysis” published in Noûs (1971), Tichý reminded the reader of his definition of intensions in (1969) and proposed his novel method of using Church’s typed λ-calculus (1940) for analysis of natural language (cf. Section 6).

### 3. Brief characterization of Tichý’s conception

Since Tichý never made a substantial change to the above picture, we may already briefly compare it with rivalling proposals.

Despite being inspired by Tractarian/Carnapian combinatorialism, Tichý does not subscribe to its contemporary Armstrong’s (1989) version (the differences between the two will be more evident from the next two sections). Since Tichý’s worlds are language independent entities, his conception of possible worlds is not *linguistic ‘ersatzism’* as, e.g., Hintikka’s (1969) conception.

Though Tichý’s views seem to contain some ideas of *abstractivism*, Tichý maintained that the mathematical nature of the combinations leads to the conclusion that *logical space*, i.e. the set of possible worlds, has to be *homogenous* (FFL 179). He thus rejected Stalnaker’s idea (1976) to set apart our actual world from the other possible worlds, which are only admitted as (useful) fictions. Modal *fictionalism* was dismissed by Tichý before it came into existence:

¹ In intensional semantics and logic, *intensions* are functions from possible worlds (Ws). Intensions include *propositions* whose values are truth-values, *properties* whose values are classes of objects, *relations* whose values are classes of n-tuples of objects, etc. That an object *instantiates* (exemplifies, possesses) a property in (or: at) a given possible world W means that the object occurs in the *extension* of that property in (at) W.
If unrealized determination systems [i.e. ‘possible worlds’] are mere fictions, then so is the realized one. (FFL 179)

Tichý’s worlds are not concrete entities, they are classes of tests and each test is distinguished from its empirical execution (compare it with a computer program as such and its concrete execution). Tichý’s conception is thus not (Lewisian) concretism. Tichý sharply rejected Lewis’ conception as an absurdity (FFL 177–80).

Lewis proposal is the only rival conception of possible worlds Tichý discussed and explicitly referred to.¹ In (1975, 91–2), he objected also to Lewis’ construal of actuality. According to Tichý, the word “actual” stands for the identity function on worlds; this explains why an addition of the word to a descriptive phrase is redundant. He maintained that we, ignorant of many facts, are not omniscient, thus we are not able (even in the future) to identify the actual possible world. Tichý repeated this idea in a number of places.

In (1971, 274–7) and also (FFL 180–3), Tichý passionately argued against varying domains and possible individuals. Without a demarcation of a domain, quantification over \( x \) cannot be logically satisfactory. The alleged examples of possible individuals (Kripke’s Pegasus, 1963) are only examples of this or that individual concept – Tichý instead used his terms individual role or individual office (an entity an individual can occupy). Tichý explicated individual offices as intensions having individuals as values. He elaborated and generalized the theory of offices to interesting heights, see (1987), which is an adaptation of his introduction to (1976). For its application see (1978d) or his analysis of Anselm’s ontological proof (1979).

Tichý was a strong antiessentialist and ‘haecceitist’ (cf. Author 2008, 2011). Though individuals do not have a genuine essence (but see Cmorej 1996, 2001), individual offices do have essences (an essence is a unique sum of defining properties called “requisites”). Tichý quantified over all individuals there are and, with the help of another variable, over all individual offices. From one viewpoint, Tichý seems to be an actualist, but from another viewpoint, he is not; however, it is less appropriate to classify his position as possibilism.

² According to Tichý (ibid.), Lewis’ worlds are not in space, since they are causally unrelated to our world. Thus, they are simply nowhere because space is by definition all-embracing. The ‘newspeak’ Lewis proposed does not help explain modality as appearing in everyday sentences.

³ The other metaphysical conception Tichý provably knew was Plantinga’s early theory (1970), but he criticized (1972) Plantinga’s essentialism.
According to Tichý, a ‘Fregean’ kind of existence is trivially applicable to any entity. He introduced (e.g. 1979, FFL, 1976) a kind of existence nontrivially applicable to offices; to say that the Pope exists in a given world W amounts to saying that the Pope-office has a holder in W. Nontrivial existence ascribable to properties (relations) says that the property has an instance in W (cf. e.g. Author 2010 for more).

4. The second and third stage: intensions, primary / derived attributes, nexuses

The second stage of Tichý’s considerations on possible worlds begins after the 1971-paper. At that time, Tichý (1974) disproved Popper’s definition of verisimilitude (likeness of theories of truth) and worked on a positive proposal he published in (1976a). This was followed by its slightly modified version in (1978d). (Tichý’s former pupil Graham Oddie wrote a whole book (1986) elaborating on and defending Tichý’s approach to truthlikeness.) Especially in (1978d), Tichý focused more on entities in intensional basis.

Tichý emphasized the difference between primary attributes which are collected in the intensional basis and derived attributes. The idea is explained further in his unpublished book (1976) where he remarked that to find out whether Xantippa is a widow one has to ascertain some more basic facts, e.g. that Socrates is dead. The idea goes back to the aforementioned Kemeny’s criticism of Carnap and it seems to be a predecessor of Kim’s idea of supervenient properties.

The second feature distinguishing the second period from the first is that intensions are not equivalent classes of procedures, but total or partial functions (as mappings) from possible worlds (cf. footnote 1).

A significant Tichý’s work of this period is an extensive unpublished monograph “Introduction to Intensional Logic” (1976) written between 1973 and 1976. The book contains the first main, atemporal, version of his logical system (see Section 6) and its applications to natural language. Its important part is a large chapter on subjunctive conditionals and related phenomena, which makes a third distinctive feature of this stage.

4 Oddie (1987) indirectly supports Tractarian inspiration for Tichýan worlds.
5 Later, Tichý wrote a paper (co-authored by Oddie, 1990) on resplicing properties over a supervenience base.
6 Tichý published three papers on subjunctive conditionals. The first one (1976b) disproves Lewis-Stalnaker’s theory. The last one (1984) disproves, among others, Tichý’s own theory from (1978). In (1984), the truth of subjunctive conditionals depends, inter alia, on a certain unarticulated, tacitly assumed parameter (such as usual nature laws).
Most of Tichý’s papers published between 1975 and 1979 are nothing but selections from this book. Needless to say that his Czech and Slovak namesakes Pavel Materna (known for his later popularization of Tichý’s logic) and Pavel Cmorej (Tichý’s former pupil who utilized his system) persuaded Tichý to adopt temporal parameter and most of his papers published at that time already contained its adoption in a rudimentary form. A typical example of a brief exposition of possible worlds in such a paper can be found in “New Theory of Subjunctive Conditionals” (1978). Note the similarity with his first stage proposal:

The aim of the investigation delimited by an epistemic framework is to determine exactly how the attributes from the intensional base are distributed through the universe of discourse at various moments of time. Before the investigation gets off the ground, the investigator faces a range of possibilities on that score. These possibilities are usually called, somewhat dramatically, possible worlds, and the totality of possible worlds is known as the logical space of the framework (Tichý 1978, 435)

A feature more characterizing the third stage is Tichý’s emphasis on nexuses (connections), i.e. higher-order relations such as cause-effect relation, as discriminating between possible worlds. He put them in intensional basis:

We thus see that a possible world is not fully described in terms of observable events such like X’s being dry, X’s being stuck, and X’s burning, taking place at definite times. What makes a world the world it is are also connections which hold between such events. Worlds differ from one another not just in what observable events take place in them, but also which events have the power bring others about. (Tichý, Oddie, 1983, 136)

However, we may find that Tichý already considered nexuses to be among the main characteristics of worlds in (1978, 435).

Thus, the only decisive feature differentiating the third stage from the second one is Tichý’s stress on temporality. In the third stage, he significantly employed verb tenses and also events/episodes, all explicated in 1980, for characterizing possible worlds.7 Here is an

7 See mainly his excellent analyses of verb tenses and temporal adverbials in (1980) and analyses of episodic verbs and verb aspects, as well as events and episodes, in (1980a). See also his philosophical defence of temporal dependence of truth (1980b).
illustration from the introductory parts of the papers which utilize temporality in explication of ability and freedom:

worlds must be allowed to branch: there must be worlds which are populated by the same individuals and whose histories are the same up to a certain time, and different after that time. (Tichý, Oddie 1983, 135)
As the possession of an attribute by an object is a time-dependent affair, the possibilities are, more particularly, possible histories of the distribution. (Oddie, Tichý 1982, 228)

5. Tichý’s late conception: determination systems

Tichý’s late conception is clearly recognizable from its stress on the idea of determination systems. These are discussed only in his masterpiece “The Foundations of Frege’s Logic” (FFL, i.e. 1988) and in (1994), which is a posthumously published introduction to his just written, but unfinished, book “Meaning Driven Grammar” which aimed to provide a highly ambitious semantic analysis of natural language. In his explanations, Tichý presupposed a reader who has only a cursory knowledge of possible worlds; he even cited his earlier relevant paper.

To introduce determination systems, Tichý deploys the notion of fact:

What are facts? The notion of fact is correlative with that of determiner. To each fact there corresponds a determiner in such a way that the fact consist either in determiner's singling out a definite object or in its failing to single out anything at all. The fact that Scott is the author of Waverley, for instance, consists in determiner A’s [= THE AUTHOR OF Waverley’s] singling out Scott. The fact that the author of Waverley is a poet consists in the proposition P’s [= THAT THE AUTHOR OF Waverley IS A POET’s] singling out [the truth-value] T. (FFL 178)

Obviously, this is nothing but Tichý’s ‘tests-on-individuals’ story. Instead of “attribute” or “office”, Tichý uses the term “determiner”; determiners are explicated (FFL 198) as possible world intensions. Instead of “test with a positive/negative outcome”, Tichý says fact; facts are actual (i.e. obtaining) or possible. Tichý explicated fact as proposition because he considered A IS GREATER THAN B and B IS SMALLER THAN B being one and the same intuitive fact.
Tichý's most elaborated view on the notion of fact appears in his re-examination of Wittgenstein's Tractatus (1994a).

Intuitive possible worlds are the total (i.e. maximal) collections of facts:

Thus the determiner's \([= \text{THE AUTHOR OF Waverley}]\) picking out George IV is a possible fact and so is its picking out nothing at all. Hence a possible world, if conceived as a totality of possible facts, will be fully characterized by an assignment of objects (of appropriate kinds) to some determiners. Let us call such assignment a determination system. A determination system is thus any many-to-one correspondence associating (some) individual determiners with individuals, (some) truth-value determiners with truth-values, etc. (FFL 178)

Briefly, a determination system specifies one combinatorial possibility as to what objects are determined (or singled out) by what intensions at what times. (FFL 199)

The determiners involved in intensional basis can be of various types:

\([\text{determiners}]\) may come classified into categories, like colours, heights, propositional attitudes, and the like. (FFL 199)

Realize that every combinatorically possible association of determiners with determinees is a determination system only relative to a given epistemic framework.

Let us assume an illustrative epistemic framework with a universe only containing \(A\) (lan) and \(B\) (arbra) and intensional basis only containing attributes (determiners) \(\text{MAN}\) and \(\text{WOMAN}\). When ignoring partiality, there are exactly 16 mappings associating the determiners with their possible values (FFL 178), i.e. 16 determination systems:

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<td>MAN</td>
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<td>WOMAN</td>
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As Tichý's explains (FFL 178–9), since determiners are not always mutually independent, not every combinatorically possible determination system is realizable. The two determiners \(\text{MAN}\) and \(\text{WOMAN}\) cannot pick out overlapping classes of individuals. Thus, not every determination system is a possible world. On the other hand, every possible world
is a determination system. Given our intensional basis and universe of discourse, only the
determination systems whose numerals are written in bold count as (intuitive) possible
worlds; we therefore have 9 possible worlds altogether.\(^8\)

Before proceeding further, I should state a few remarks explaining Tichý’s position.
The possible world no. 16 seems to be a suspicious empty world. However, this idea is
mistaken. Firstly realize that Tichý’s worlds are not individuated by a certain population of
individuals.\(^9\) The possible world no. 16 is simply a world in which \(A\) and \(B\) are tested on
instantiation of the properties \textit{man} and \textit{woman} with a negative result. The individuals have
different properties than the two primary ones contained in our miniature intensional
basis, e.g. the property \textit{be not a man}.\(^{10}\)

Unrealizable determination systems in some ways resemble impossible possible worlds
which are discussed so much today. Impossible worlds are proposed for a variety of
reasons,\(^{11}\) none of them are important for Tichý. For instance, he prevented the problem of
logical omniscience by employing hyperintensional entities (cf. Section 6). Recall also that
Tichý proposed possible worlds in connection with the idea of subject investigating the
external, empirical reality. Both empty and impossible worlds are beyond such
considerations.

Surprisingly, Tichý did not employ and develop in FFL an idea which suggests itself,
viz. that a determination system is only a single ‘slice’ (or ‘time point’, Kuchyňka, Author
2014) of a possible world, which is thus a sequence of such slices. An anticipation of this idea
can be found in (Oddie, Tichý 1982, 228, cf. the quotation above), its full expression is this:

\begin{quote}
we must think of it [i.e. possible world] as a world \textit{history}, a whole course of events
unfolding in time. In other words, a world must be conceived of not as a single
\end{quote}

\(^8\) In (FFL 179), Tichý speaks about 8 worlds but, when discussing an entirely analogous example in (1994, 60),
he mentioned the number 9, which is, I maintain, the correct one.

\(^9\) Vladimír Svoboda (e.g. 2001) imagined that individuals may escape ‘our world’, moving thus to the world no.
16, by losing any (remarkable) property and reaching a kind of ‘limbo’. A closer examination of this idea (e.g.
Author 2008b) leads to the conclusion that such proposal contradicts Tichý.

\(^{10}\) Some metaphysicians reject ‘negative’ properties but Tichý belongs to the opposite camp: \textit{be a non-ferrous},
for instance, is a good property the possession of which can be sensibly empirically tested. When properties
are explicated as possible world intensions (cf. Author 2011, I.2) any trace of negativity evaporates, because no
intension displays any mark of negation.

\(^{11}\) Cf., e.g., Berto (2013), Vacek (2013).
distribution of the traits from the intensional base through the universe of discourse, but as a series of such distributions, one for each moment of time (Tichý 1994, 62)

So far, we have an intuitive notion of possible world as an entity which contains facts whereas a fact consists in that a determiner picks out a determinee. What then are the logically simple entities $W_1, W_2, ..., W_n$, which are the arguments of Tichý’s intensions?

Oddie suggested to call these Ws *proto-worlds* (1986, 125) and assumed their correlation with the ‘thick’ possible worlds. In (FFL 194–200), Tichý enlightened us of the link between the two kinds of worlds as follows.

Firstly recall Carnap’s notion of explication: it consists in the replacing of an intuitive notion – here the notion of possible world as a collection of facts – to its rigorous mate, its explicatum. Now there is a small complication. Tichý explicated facts as possible world propositions, which are classes of world/time couples. As noted already by Stalnaker (1976) and even Adams (1974), both not mentioned by Tichý, there is a question how can possible worlds be classes of propositions when propositions are classes of worlds (or world/time couples).¹²

Tichý realized this *circularity problem* (FFL 194) and solved it by carefully distinguishing between logically primitive and complex entities and their role in explication.¹³ For Tichý, propositions are the primary goal of his investigation, thus possible worlds must be taken as logically primitive, while propositions will then be defined in terms of worlds.

But to explain anything, propositions must be tied into the system of explication to the intuitive notions because it does matter whether a particular class of world/time couples explicates the intuitive proposition *Alan is a man* or rather *Alan is not a man*. Such a link of a proposition to its intuitive correlate is guaranteed by an *interpretation* of the members of $\omega$ (i.e. collection of Ws) and the two truth-values. Interpretation is obviously a reverse of explication:

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¹² Oddie (1986, 125) considered a slightly different puzzle: possible worlds have to be specified even by some functions defined on worlds (e.g. nexuses between propositions); but how then can one specify a single world? Oddie’s solution anticipated Tichý’s.

¹³ A logically primitive entity differs from another entity of the same sort only by its different numeric identity.
Now to interpret the basic category $\omega$ is to assign to each of its members a unique determination system. ... The determination systems which are assigned, within the [given epistemic] framework, to $\omega$-objects are called the possible worlds of the framework. (FFL 199-200)

Let us briefly reflect on Tichý’s position. Tichý’s contribution to metaphysics consists in explaining possible worlds as determinations systems. This is a refinement of the idea that possible worlds are total classes of compatible facts.\textsuperscript{14} For logical investigations, however, Tichý decided to use only surrogates of these worlds, namely the logically primitive entities $W_1$, $W_2$, ..., $W_n$, which are interpreted by the possible worlds as determination systems (or rather their chronologies).

A metaphysician can be perhaps a bit uncomfortable with the fact that Tichý’s framework, used by him and others for a number of useful explications of various notions, does not provide a ‘full blooded’ explication of the notion of possible world. Yet there is a way out of this suggested by the present author (e.g. 2009a) who utilized ideas and apparatus from (2008a, 2011 with Kuchyňka).\textsuperscript{15} We can retain Tichý’s framework as it is, with all those explications, but we must rethink the role of Ws. I suggest understanding Ws as a mere modal factor. A proposition, for instance, is clearly a function whose values depend on logical modality (modal factor). Possible worlds are then explicated as some other, complex entities which reflect the intuitive features of classes of facts, whereas facts are some structured entities, not possible world propositions.

In the next section, I am going to briefly explain the role of possible worlds (as Ws) in Tichý’s semantics and logic.

### 6. Possible worlds in Tichý’s Transparent intensional logic

An admissible introduction to Tichý’s semantics likens it to Montague’s semantics. Though the comparison is rather coarse and misleading in various respects, it can be used as a starting point.

\textsuperscript{14} Which goes back to C.I. Lewis (1923).

\textsuperscript{15} The original motivation is the present author’s solution (2007, 2014 with Kuchyňka) to the puzzle studied already by Petr Kolář (e.g. 2002): Tichý rejected the idea of facts as structured entities by evoking the famous aRb fact; but then, there is the problem of how to preserve correspondence theory of truth which is based on some sort of isomorphism between sentences and their significance (facts).
Montague published his main papers around 1970 (cf. his posthumous collection 1974). Tichý referred to his work firstly in (1971) and it seems that he became acknowledged with Montague’s results after developing his own system.

Tichý’s system is a straightforward adaptation of Church’s \( \lambda \)-language (except constants and variables for any objects, one has applications of functions to arguments and \( \lambda \)-abstractions representing unapplied functions). \(^{16}\) Tichý enriched Church’s type basis containing \( \omega \) (two truth-values) and \( \iota \) (individuals) by the type \( \omega \). Over such basis, there are numerous functions of composite types. For instance, the type of propositions is \( (\omega\omega) \) (in another notation: \( \omega \to \omega \)). After (1971), Tichý also adopted partial functions, thus some intensions can be without a value (gappy). In the late 1970s, he also admitted the temporal parameter, \( \tau \). Chronologies of \( \xi \)-objects (for arbitrary type \( \xi \)) are then functions of type \( (\xi\tau) \).

Intensions are functions from possible worlds to chronologies, their type is \( ((\xi\tau)\omega) \). (For the sake of brevity, I will suppress the temporal parameter below.)

Montague introduced his system as a modification of extensional logic and the implementation of intensions corresponds to their supplementary role because he used them only for opaque (intensional) contexts. Tichý, however, assigned intensions to expressions having modally conditioned reference in every context, including the transparent one; this is why he called his system Transparent intensional logic.

But even Tichý’s technical treatment of intensions is distinct from Montague’s (cf. e.g. Tichý 1978b for comparison of the two systems). Quite in the spirit of Church’s system, Tichý’s terms standing for intensions are \( \lambda \)-abstractions over possible worlds. Such \( \lambda \)-abstraction \( \lambda w(...w...) \) can be combined with variable \( w \) ranging over possible worlds in order to express a recourse to a value of that intension in (the value of) \( w \). Thus, \( (\lambda w(...w...) \) \( w \) yields a value of the intension represented by \( \lambda w(...w...) \), while \( \lambda w(...w...) \) represents the intension as such (cf. e.g. Tichý 1978c). In (1976), Tichý introduced also a deduction system which was published in 1980s (cf. the relevant papers in 2004).

\( \lambda \) is a variable binding operator and \( \lambda \)-formalism is capable of expressing subtle differences in scope. To illustrate, \( \lambda w(...w...w') \) stands for an intension in \( \lambda w... \lambda w'(...w...w')... \), despite it is yielded with a partial dependence on the value of \( w \). This enables a logically lucid treatment of de re / de dicto propositional attitudes as well as scopes of modal

\(^{16}\) \( \lambda x(...x...) \) stands for a function of \( x \); \( \lambda \)-abstractions of form \( \lambda x(F x) \) are \( \eta \)-reducible to \( F \). (\( F x \)) or (\( f a \)) are applications.
operators. Let us add that modal operators are classes of propositions, being of type \( \varnothing(\varnothing w) \). They are definable by means of quantifications over possible worlds, e.g., \((\varnothing p) \iff (\exists \lambda w(p \circ w))\). Famous puzzles concerning modality are quickly solvable (cf. Tichý 2004, Author 2009).

In (1970), Lewis published a certain criticism of intensional (or possible world) semantics. At the time, intensional semantics was at the beginning of its noteworthy development during 1970s and early 1980s and the force of the argument was only appreciated afterwards. The argument roughly says that possible world intensions are too coarse-grained to be meanings of expressions. In other words, intuitive meanings have a more fine-grained structure, thus we need hyperintensional entities which stand in semantic scheme between expressions and extensions/intensions signified by them. For another argument, when \( X \) believes that \( 1+1=2 \), we can hardly entail that \( X \) believes Fermat’s Last theorem, despite that “\( 1+1=2 \)” and the famous theorem stand for one and the same proposition which is true in all possible worlds.

After (1971), Tichý found a solution when he noticed that his \( \lambda \)-terms can be read in an extensional way as representing usual set-theoretic objects, e.g. possible world intensions, or they can be read in ‘intensional’ way as representing entities which determine the set-theoretic objects. Tichý called those ‘intensional’ entities “constructions” and he later explained (1986) that he borrowed the term from geometry where a point or circle can be constructed one way or another. Constructions are structured, abstract, extra-linguistic entities. They differ from \( \lambda \)-terms usually used for their records. They are akin to algorithmic computations (Tichý 1986, 526). Each construction is specified by the object \( O \) it constructs and the way it constructs \( O \). One object \( O \) is constructed by an infinite number of constructions which are thus equivalent (congruent), despite being non-identical. For specification of kinds of constructions and their defence see esp. FFL.

Constructions are suggested by Tichý as explicata of meanings already in (1976). His semantic scheme is entirely Churchian (‘Fregean’): an expression \( E \) expresses (in \( L \)) a construction \( C \) (= the meaning of \( E \)) whereas \( C \) constructs an intension or extension (= the significance of \( E \)). For instance, the meaning of the sentence “Fido is a dog” is the construction (simplifying the notation) \( \lambda w(\text{Dog}_w \circ \text{Fido}) \), which constructs the proposition \( \text{FIDO IS A DOG} \). Propositional attitudes are explicated in FFL as attitudes towards propositional constructions, not towards mere propositions; this proposal blocks undesirable
consequences embraced by intensional semantics and solves the problem of *logical omniscience*.

Tichý’s special kind of entities, constructions, can be used for capturing features of intuitive entities that cannot be explicated by means of mere (possible world) intensions. To give at least one example, we can suggest that a fact is a propositional construction and that possible worlds are classes of such structured facts (Author 2007).

The type of constructions is split; Tichý’s late system is thus a special *ramified theory of types*. Consequently, it is easily capable of avoiding a number of *paradoxes concerning ‘propositions’* (propositions not in the strict sense of possible world semantics). One of the best known paradoxes concerning possible worlds and ‘propositions’ is Kaplan’s (1994). In his section “42. Limitation of Logical Space” (FFL 218 and passim), Tichý reminded the reader of the fact proven by Cantor that there are more mappings from (nonempty) set $S$ to (nonempty) set $S'$ than there are members of $S$. Thus, there are more propositions than worlds: no mapping can associate, in 1-1 fashion, every proposition with a certain world (or world/time couple). Consequently, no matter how one explicates relations such as belief, assertion, ... between individuals and propositions, “there will always be a proposition such that at no world/time couple is it the only proposition believed, or asserted, by George IV.” (FFL 219). Tichý demonstrated the situation and immediately proved Theorem 42.1, the corollaries of which provide his solution to the *Liar paradox* (which bears resemblance to his solution in 1976).

7. Conclusions

To repeat the most essential features of Tichý’s conception of possible worlds, Tichý proposed it in close connection with his sophisticated and extensive logical system. Similarly, as other intensional logicians/semanticists, he used possible worlds for analysis of natural language and also further explication of various notions of our conceptual scheme such as fact, causality, event, ability, freedom, propositional attitudes, subjunctive conditionals and modalities. Though it is problematic to subsume his conception under the known ones, “combinatorialism” and “actualism” seem to be the most appropriate labels.

Tichý’s first proposal was published in 1968. His conception underwent various modifications, but its main character remained unchanged. In the first stage, possible worlds were classes of certain procedures (algorithms) consisting mainly of tests on
individuals. In the second and subsequent stages, they were intuitively total classes of facts but Tichý technically treated them as arguments for his intensions, i.e. as logically primitive entities. The combinatorial character was underlined by his late conception in which the intuitive possible worlds are explicitly specified as some determination systems; determiners are then explicates as intensions.

Tichý usually stressed intensional dependence of attributes/determiners and also the idea that worlds are individuated even by the nexuses which are realized in them. In the late 1970s, he added temporality. Tichý always preferred fixed domain; he modelled ‘possibilia’ as individual offices, i.e. some intensions. Since his recent type framework is explicitly ramified, his approach can avoid various paradoxes concerning possible worlds and ‘propositions’.

Some questions not answered by Tichý, but posed and answered by his followers, concerning the structured nature of facts, the nature of correspondence, and the possibility to construct possible worlds from structured facts.

References


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18 Tichý’s papers are reprinted in Tichý (2004).


