On Interaction of Semantics and Deduction in Transparent Intensional Logic (Is Tichý's Logic a Logic?)

Jiří Raclavský

Abstract

It is sometimes objected that Tichý's logic is not logic because it underestimates deduction, provides only logical analyses of expressions. I argue that this opinion is wrong. First of all, detection of valid arguments, which are formulated in a language, needs logical analysis ascertaining which semantical entities, Tichý's so-called constructions, are involved. Entailment is defined as an extralinguistic affair relating those constructions. Validity of arguments, composed of propositional constructions, stems from properties of constructions. Such properties are displayed by derivation rules of Tichý's system of deduction.

Keywords: logical analysis; deduction; entailment; Transparent intensional logic.

1 Introduction

Tichý's logic or, more narrowly, *Transparent intensional logic*, is often thought to be difficult to classify, since it is not a logic in the usual sense of the word. By logic one usually understands certain deduction system (or perhaps their class); only sometimes logic is understood in the sense of certain analytical method or something in this sense. If a logic is the latter, there is a doubt why it is not the former.

Some even say that in Tichý's logic the analytical component predominates the deductive component. Sometimes such remark suggests that Tichý's logic neglects something very important for any logic.[1] It subsequently seems that the research within the framework of Tichý's logic avoids what makes a logic to be a logic – viz. deduction, investigations of what we can validly infer from what.

The aim of this paper is to show that such view on Tichý's logic is misleading. I will argue that the very project of Tichý's logic entertains

[1] I have met such opinion several times not only in personal communication but even in print. For instance, the opinion about the supremacy of analysis over deduction in Tichý's logic was expressed by Karel Šebela (2006).

deduction in a much larger extent than it seems to some at first glance. In other words, Tichý's logic really is a logic.

The core of my justification is in fact simple and generally known. If the aim of logic is to detect valid arguments which are formulated in language, it is firstly inevitable to detect what exactly the sentences of the argument mean, their exact meaning. In other words, *logical analysis* (or logical semantics) of language expressions is a substantial service for logic, incl. deduction. (Needless to say that this is in accordance with the view that meanings of sentences, i.e. 'thoughts', are primary, thus the entailment among the meanings of sentences is primary. If this is clarified, and it is also clarified which expressions of this or that language express those 'thoughts', we are also ready to determine valid arguments in their verbal, language formulation.)

Here, we can also reproduce Tichý's unpublished opinion "if we will know, what we are talking about [i.e. what the meanings of our expressions are], we will also know what entails what" (cited in Štěpán, 55). This opinion is stronger and admittedly more controversial than that from the preceding paragraph. But I will attempt to show that it is very much the point.

In the rest of the paper I will gradually proceed from questions related to logical analysis to deduction – "2. Relationship of logical analysis to entailment", "3. Relationship of entailment to deduction". Some completion will be provided by the sections "4. Derivation systems" and "5. Concluding remarks".

2 Relationship of logical analysis to entailment

First, I will introduce basic notions of Tichý's semantics, but I will avoid the details. I will not, for instance, explain how and why exactly Tichý models meanings of expressions in a particular hyperintensional way; I will simply recapitulate that it is so. The first part of this section focuses on notions related to logical analysis, the second part on the notion of entailment.

According to Tichý, *extensions* or *intensions* (which are functions from possible worlds w and moments of time t) are *denotata* of expressions. For instance, sentences denote *propositions*, i.e. total or partial intensions having truth-values (T or F) as their functional values.

Any object is constructed by infinitely many non-identical but *equivalent* constructions. *Constructions* are abstract, and also extra-linguistic, structured procedures. They have an algorithmic character; they are not set-theoretical objects (though they usually construct set-theoretical objects). Constructions are specified by *which* objects they construct and *how* they do so. Basic kinds of constructions can be understood as objectual correlates of λ-terms. These kinds are: variables (which are of form) x (the corresponding λ-terms are variables as letters), trivializations ${}^0X$ ('constants'; X is any

object or construction), compositions $[CC_1 \ldots C_n]$ ('applications'; C or C_i is any construction), closures λxC ('λ-abstractions'). Constructions always construct an object of a particular type (e.g. of the type of propositions); *cf.* Tichý's theory of types.

Constructions can be aptly considered to be explicantia of language *meanings* which are *expressed* by language expressions. Thus, expressions *mean* constructions, which are their *logical analyses*.[2] I maintain that semantic notions are relative to language.[3] Thus, an expression e expresses in a language l the construction c which constructs the denotatum of e in l. The semantic scheme is this:[4]

expression e
— e expresses in l:
construction c, i.e. the meaning of e in language l, the logical analysis of e
— the construction c constructs, the expression e denotes in l:
intension/non-intension, i.e. the denotatum of the expression e in l

Tichý's *semantics* is *hyperintensional* in the sense that its individuation of meanings is finer than that by intensional semantics, due to which meanings of expression are simply intensions or extensions. Contemporary literature states many reasons why adopt hyperintensional individuation of meanings.[5]

Logical analyses of sentences are inevitable for examination of validity of arguments made from them. A correct argument formulated in language is such that its conclusion follows from its premises. This is, however, a language relative affair.

The common definition of entailment (viz. a class of sentences $s_1, \ldots, s_n$entails a sentence-formula s iff ...) is inadequate. It is so because of an unwarranted ignorance of the fact that expressions, formulas being no exception, are language relative in the sense that they have certain meaning in one language ('notation'), while having another meaning (or no meaning at all) in another language. Thus a sentence-formula can be entailed by some class of sentences-formulas in one language, yet not in another.

Entailment among sentences is dependent on entailment among entities which are meant by those sentences in this or that language. Therefore, a definition of language-entailment should rather be[6]

[2]To provide an explicans of meaning of certain expression in a given language does not amount to providing its translation to some (possibly formal) language, i.e. offering merely another expression; *cf.* Raclavský 2010.

[3]For my explication of language and semantic notions, see Raclavský 2009 (ch. IV.5).

[4](Raclavský 2009, 63).

[5]For this and related topics see especially the first chapter of Raclavský 2009 or Duží, Jespersen and Materna 2010.

[6](Raclavský 2009, 264).

> a class of sentences $s_1, \ldots, s_n$ *entails in* language l a sentence s iff what (viz. a class of constructions) is expressed by $s_1, \ldots, s_n$in l entails* what (viz. a construction) is expressed by s in l.

And, of course,

> a language argument a is *valid in* language l iff (the class of) a's premises $s_1, \ldots, s_n$ entail, in l, a's conclusion s.

The definition just given mentions entailment* among constructions, which needs a clarification. Since entailment is a topic for a specialized study, here I can only list the most essential findings.

We could firstly define entailment among propositions and the entailment among propositional constructions will be defined as dependent on it. Though I omit this here, there will still remain a linkage to the former notion thanks to the truehood of constructions which is dependent on the truehood of propositions (*cf.* Raclavský 2009, 348-351):

> a construction c^k is *true* in* w at t iff there exists a truth-value o such that o is the value of that what (viz. a proposition) is constructed by c^k in w at t and o is identical with the truth-value T.

Thus,[7]

> a class of constructions ${c^k}_1, \ldots, {c^k}_n$ *entails** a construction c^k iff for every w and t it holds that if ${c^k}_1, \ldots, {c^k}_n$ are true* in w at t, then c^k is also true* in w at t.

Concluding: logical analyses (i.e. constructions) of expressions are closely and indubitably related to entailment; *if we know what sentences mean (i.e. which constructions) we are capable to determine what entails what.* Now there is a question – how does it relate to deduction?

3 Relationship of entailment to deduction

Tichý's deduction system[8] is not so well known, but I profess it. I will introduce here, though in a simplified manner, its basic notions. I will show then how entailment relates to deduction. (Both entailment and deduction

[7] (Raclavský 2009, 160).

[8] *Cf.* especially Tichý 1982, which is a compressed core of deduction from Tichý 1986. A substantial elaboration of substitutability of variables brings the condensed study Tichý 1986. Various contributions relevant to deduction can be found also in Oddie and Tichý 1982).

are understood in an objectual sense; the way to proceed from the objectual level to the linguistic level was shown in the preceding section.)

A *match* M is a couple

$$X{:}C,$$

where C is a construction and X is the trivialization of an object of type ξ or a variable ranging over objects of type ξ. We will say that a match is *satisfied* by a valuation, which means that the construction C constructs on that valuation the very same object as the construction X. A *sequent*

$$\Phi \Rightarrow M$$

has two members; Φ is a class of matches and M is a match. A sequent is *valid* if every valuation which satisfies all members of Φ satisfies also M. A *derivation rule*, in Tichý's earlier terminology *inferential rule*, is a validity preserving operation on sequents. It is of form

$$\Phi_1{\Rightarrow}M_1;\ \Phi_2{\Rightarrow}M_2;\ \ldots;\ \Phi_n{\Rightarrow}M_n \models \Phi{\Rightarrow}M,$$

Its final sequent $\Phi{\Rightarrow}M$ is valid when all the sequents $\Phi_1{\Rightarrow}M_1$, $\Phi_2 \Rightarrow M_2$, $\ldots$, $\Phi_n{\Rightarrow}M_n$ are valid.

Let me explain what happens here. Matches can be understood as certain identity statements – a particular match thus puts the identity relation between an object O and the result of constructing of certain construction C, which is that object O.[9]

Now let p or p_i be a variable for propositions, C^π or ${C^\pi}_i$ be a construction of a proposition. Sequents such as

$$\models \{\ p_1 : C_1^\pi,\ p_2 : C_2^\pi,\ \ldots,\ p_n : C_n^\pi\} \Rightarrow p : C^\pi$$

can be *construed* as certain implications holding between the conjunctive connection of these constructions (matches) and the final propositional construction (match).[10] Arguments understood in an objectual way (i.e. not their verbal, linguistic formulations) can be *represented* just by the sequents of form

[9] I mean constructions of the form $[{}^0O^0{=}[{}^0\Gamma^\xi C]]$ (alternatively: $\lambda w \lambda t[{}^0O^0{=}[{}^0\Gamma^\xi C]]$), where ${}^0\Gamma^\xi$ constructs the partial function which maps constructions to the ξ-objects (if any) constructed by them (remark: a use of so-called double execution, *cf.* Tichý 1988, would be more appropriate here). Trivializations of well-known logical functions are written in an infix way.

[10] Since implication (material conditional) operates on truth-values, we have to manage a way from the constructions-matches to propositions, or rather their values. It is not technically complicated to implement it but it is omitted here.

(From a more common viewpoint, the validity of the argument is *measured* by that rule.)

Thus, properties of propositional constructions (or: which properties are possessed by the propositional constructions) determine the entailment (which classes of constructions entail* which constructions), i.e. determine the transfer of validity by means of the respective rule of derivation.[11] So this is how I understand the relationship of deduction and entailment.

(For completeness, let us also introduce Tichý's construal of deriving. A sequent is *derivable* from a class of sequents *according to* a *derivatiĕon rule*. A finite sequence of sequents is called a *derivation with respect to class R of derivation rules*, writing it $\vdash_R \Phi \Rightarrow M$, if every item of that sequence, i.e. a *derivation step*, is derivable from the preceding steps according to some derivation rule from R. Note that this guarantees the relevance of that movement on the derived step. Realize also that Tichý construed *inference* as sequence of (valid) arguments, i.e. sequence of logical truths. Tichý vehemently criticized the so-called inference from assumptions, which are not logical truths.[12])

Let us put a closer look on an important property of certain derivation rules. Bi-directional derivation rules

$$\models x{:}C_1 \Leftrightarrow x{:}C_2$$

(which is a shortcut for $\models \{x{:}C_1\} \Rightarrow x{:}C_2$ and $\models \{x{:}C_2\} \Rightarrow x{:}C_1$) elucidate which object is constructed by the construction C_1 (and C_2). Thus, they also elucidate which particular construction it is.

In some cases, one of the two constructions C_1 and C_2 is significantly simple, consider 'transformation of disjunction to implication' as an example

$$\models f{:}\lambda o_1 o_2[o_1^0 \vee o_2] \Leftrightarrow f{:}\lambda o_1 o_2[[{}^0\neg o_1]^0 \rightarrow o_2],$$

(where f is a variable for binary truth-function, o_i a variable for truth-values).[13] I consider this sort of rules to be *definitions* as it satisfies many in-

[11] Notice also that sequents or derivations concerning propositional constructions (*cf.* the above example) are only a special case of what can be treated by Tichý's system of deduction. In Tichý's system one can also work with sequents concerning, e.g., classes of numbers, etc. Tichý thus substantially expands the field for deduction (already Jan Štěpán noticed that in Materna and Štěpán 2000, 106).

[12] *Cf.* Tichý 1988 (chapter Inference) or Tichý and Tichý 1999.

[13] The construction $\lambda o_1 o_2[o_1^0 \vee o_2]$ is η-reducible to ${}^0\vee$. Thanks to some other rule, the definition can be stated in the form $\models o{:}[o_1^0 \vee o_2] \Leftrightarrow o{:}[[{}^0\neg o_1]^0 \rightarrow o_2]$.

tuitions concerning definitions (Raclavský 2009, 287-290). A definition says that the equivalent of $\lambda o_1 o_2[o_1^0 \vee o_2]$, i.e. ${}^0\vee$, is the construction $\lambda o_1 o_2[[{}^0\neg o_1]\ {}^0\rightarrow o_2]$; in other words, the definition elucidates which object, which truth-function, is constructed by the construction ${}^0\vee$. The definition does not 'create' a 'new' construction ${}^0\vee$; the construction ${}^0\vee$ has already been there before the definition. The definition only makes clear which object is constructed by ${}^0\vee$ and this way it makes also clear how ${}^0\vee$ relates to the (equivalent) construction $\lambda o_1 o_2[[{}^0\neg o_1]{}^0\rightarrow o_2]$. To state the point a bit differently, the validity of the sequent (definition) is given especially by the fact which particular objects are constructed by ${}^0\vee$, ${}^0\neg$, ${}^0\rightarrow$.

So-called inferential semantics (and also the theory of implicit definition or of 'defining' theory) is based on the intuition that the meaning of an unknown but just introduced operator will be set by showing its inferential relations. Already A. N. Prior remarked, during the discussion concerning the operator "tonk", that the meaning of an operator had already been there before its introduction; inferential rules only show exactly which particular meaning it is. Tichý would surely subscribe to such view, it is fully in the spirit of his approach. Setting now definitions aside, there are many other derivation rules which show (practically manifest) properties of objects. For instance, one of the properties of implication – viz. that it returns the truth-value T for ⟨T,T⟩ – is exhibited by the rule

$$\Phi \cup \{{}^0\mathrm{T}{:}o_1\} \Rightarrow {}^0\mathrm{T}{:}o_2 \models \Phi \Rightarrow {}^0\mathrm{T}{:}[o_1{}^0\rightarrow o_2].$$ [14]

Let us summarize what has appeared repeatedly in this section: *if we know what the meaning of a certain expression is, i.e. which construction it is (thus we also know which object is constructed by it), we are also capable to determine valid arguments which can be understood as valid inference rules.*

3.1 Derivation systems

Still, some might object even now that Tichý's logic is not a logic in the proper sense, referring here to calculi, completeness and similar things. In order to also explain this matter from the viewpoint of Tichý's logic, let us consider a class of construction *CS* and a class of derivation rules *R*. By a *derivation system* I will understand the couple

$$\langle CS,\ R\rangle.$$ [15]

[14] Many such rules, though not just this one, were showed by Tichý and Oddie in their paper ([3], 214).

[15] This is in fact only a rudimentary form of derivation systems (they were first introduced in Raclavský 2009). The present construal, elaborated together with Petr Kuchyňka Raclavský and Kuchyňka 2011, is a bit more complex.

For the sake of illustration we will utilize the example of propositional logic (PL).

Classical propositional logic (CPL) operates on a certain *area* of objects, namely two truth-values and n-ary total truth-functions. But the *subject matter* of CPL ('aboutness') consists of certain constructions of objects from the objectual area. Once more: the subject matter of CPL is not made of the objects, but rather certain constructions of those objects. In our case, the constructions comprise i. variables for truth-values (o, o_1, ..., o_n, i.e. familiar ps and qs), ii. trivializations of the truth-functions (${}^0\neg$, ..., ${}^0\vee$, ${}^0\rightarrow$, ...), and also iii. compositions of constructions from i. and ii., e.g. the construction $[{o_1}^0 \rightarrow o_2]$. Note that CPL is not about constructions of the form closure, e.g. $\lambda o_1 o_2 [[^0\neg o_1]^0 \rightarrow o_2]$; neither is CPL about variables, say f, for the truth-functions; it is not about various constructions built from such constructions and constructions from i.-iii. Now I am going to show that Tichý's system of deduction can treat them, it is capable to treat all constructions which can be considered to be in the subject matter of PL.

For CPL, there exist a number of calculi in our construal, certain derivation systems.[16] As so-called axioms, one can choose some 'tautological' constructions, e.g. $[{o_1}^0 \rightarrow [{o_2}^0 \rightarrow o_1]]$. (Alternatively, axioms can be understood as categorical rules.) They form a class ACS_{CPL}, which is a subclass of CS_{CPL}. It is obvious that with help of derivation rules from R_{CPL}, e.g. $\models \Phi \cup o{:}[{o_1}^0 \rightarrow o_2]$, $o{:}o_1 \Rightarrow o{:}o_2$ (i.e. modus ponens), one can reach (by deriving) all 'tautological' constructions from the difference of classes CS_{CPL} and ACS_{CPL}, which amounts to the completeness of this particular DS_{CPL}.

(It is well-known that calculi for PL work exclusively with some 'connectives'. It means that the calculi operate only within a part of DS_{CPL} which has been considered above because the calculi do not allow, for instance, constructions containing ${}^0\vee$ or they 'introduce' them by mean of definitions, which means – as I take it – that they utilize rules such as $\models o{:}[{o_1}^0 \vee o_2] \Leftrightarrow o{:}[[^0\neg o_1]^0 \rightarrow o_2]$.)

Yet this is only a fragment of what PL is from my viewpoint. If we look on this occasion at Tichý's papers on deduction we can notice that Tichý considered not only constructions of various kinds but mainly a lot of various rules. Thus particular derivation systems have to be extracted or selected from Tichý's writings. We can then study derivation in, say, 'quantified' PL. In derivation systems for such logic the class of construction CS is similar to that for CPL, but containing certain constructions of kind closure and also their compositions with ${}^0\forall$ or ${}^0\exists$. For instance, $[^0\forall \lambda o_1 o_2 [[^0\neg [{o_1}^0 \vee o_2]]^0 \rightarrow [[^0\neg o_1]^0 \wedge [^0\neg o_2]]]]$, which is in fact De Morgan's

[16] Generally, DS is best given by explicit determination of all primitive constructions of its CS. In the case of (objectually understood) calculi – though they are DSs –, it is more fundamental to determine its composed 'tautological" constructions (i.e. constructions constructing the truth-value T on any valuation).

law, is surely something PL can be about. For other examples of PL: admitting variables for truth-functions we get PL of 'higher' order; allowing quantification over constructions of PL-objects we get even a real higher-order logic (in the sense of Tichý's theory of types). And I do not even mention the possibility to accept constructions of partial functions, as Tichý always did.

If we would like to work within modal PL, we will admit into the appropriate derivation systems variables for propositions, possible worlds, moments of time ($p, \ldots, w, \ldots, t, \ldots$), and the appropriate quantifiers (having thus also operators of necessity and possibility). Then, one of the fruitful contributions done by Tichý is a sophisticated treatment of substitutivity of variables such as o by means of constructions such as p_{wt} (this way it is possible to correct classical rules of, say, PL, in order to keep validity, which is generally lost in frameworks which adopt partiality). We can continue with the enrichment of CPL further and further. Tichý thus provided extensive and at the same time unified framework of deduction. (That this framework is not investigated enough is another thing – it is rather a task for the future.)

Since it relates to non-empirical matters, the example with PL does not illustrate enough one important feature of derivation systems. Rules of Tichý's system of deduction can be roughly classified in three groups: 1. 'basic' ones (if one does not know them, one understands nothing), 2. 'displaying' (displaying, for instance, particular properties of implication) and 3. 'content' rules (definitions). It is not only the rules of kind 2. can be acquired from the analytical cognition of an object. Content rules, 3., are perhaps more interesting in that. Their exemplary use can be found in Tichý's and Oddie's study on logic of ability, freedom and responsibility [3]. In that paper, the rules of kind 1. and 2. are introduced as first and the derivation system DS is then gradually enriched by allowing other and other especially content rules which concern the notions of ability, freedom and responsibility.

3.2 Concluding remarks

In conformity with the current logical methodology, numerous axiomatic systems, or logics, are proposed; their role is twofold. Firstly, their task is to define implicitly some key notions (or objects). A particular system of modal logic, for instance, should specify the notion of the property 'being a necessary proposition' (i.e. to define the meaning of the 'box'). Secondly, their task is to lay out a certain derivation system in which the deduction with that notion takes place.

We have already seen that the first task is superfluous from Tichý's viewpoint. Tichý even made a remark in the sense that the very idea of logic presupposes that the entities for which an axiomatic system is proposed

exist prior to that axiomatization (*cf.* Tichý 1988, 277).

As regards the second task, recall that those logics are proposed only for thematically narrow fields; for instance, deontic logics only investigate a few notions related to norms. But the research within Tichý's logic concerns a rather great number of subjects. We can put them in categories such as 'logic of propositional attitudes', 'logic of subjunctive conditionals', 'temporal logic', etc.. Realize also that the ambition of Tichý's logic is to work on the unified framework. It thus cannot happen that the results of, say, Tichý's 'temporal logic' would be incompatible with Tichý's logic of 'propositional attitudes'.

I do not claim that for all those 'sublogics' of Tichý's logic concrete derivation systems have been already built. Such things are task for the future. On the other hand, realize that Tichý offered a number of derivation rules which can be utilized in those particular 'logics'. Recall also that discovering rules goes hand in hand with an adequate analysis or, more precisely, with the proper explication of relevant intuitive notions.

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