Explicating the Notion of Truth within Transparent Intensional Logic

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Abstract

The approach of Transparent Intensional Logic to truth differs significantly from rivalling approaches. The notion of truth is explicated by a three-level system of notions whereas the upper-level notions depend on the lower-level ones. Truth of possible world propositions lies in the bottom. Truth of hyperintensional entities – called constructions – which determine propositions is dependent on it. Truth of expressions depends on truth of their meanings; the meanings are explicated as constructions. The approach thus adopts a particular hyperintensional theory of meanings; truth of extralinguistic items is taken as primary. Truth of expressions is also dependent, either explicitly or implicitly, on language (its notion is thus also explicated within the approach). On each level, strong and weak variants of the notions are distinguished because the approach employs the Principle of Bivalence which adopts partiality. Since the formation of functions and constructions is non-circular, the system is framed within a ramified type theory having foundations in simple theory of types. The explication is immune to all forms of the Liar paradox. The definitions of notions of truth provided here are derivation rules of Pavel Tichý's system of deduction.

1 Introduction

I suggest an explication of the notion *true* within the extensive logical framework of Pavel Tichý's *Transparent Intensional Logic* (*TIL*). The approach differs significantly from other well-known approaches to truth such as the hierarchical and bivalent proposal by Tarski (1933/1956), three-valued theories by Kripke (1975) and others, paraconsistent dialetheism by Priest (1987), revision theory by Gupta and Belnap (1993), paracompleteness by Field (2008) and Beall (2009), axiomatic approaches by Halbach (2011) and Horsten (2011), etc. The brevity of space does not allow to provide a comparison of the present approach with the aforementioned ones; nevertheless, some differences can be read from the rest of this introduction and some other remarks in the paper.

The key feature of the present approach is that the truth of certain non-linguistic entities is construed as primary, while the truth of linguistic entities, which represent the non-linguistic ones, is defined as dependent on it.

The notion of truth, as explicated in TIL, splits in three kinds according to the range of their applicability to:

a. propositions (which can be considered to be denotata of expressions),

b. (so-called) constructions of propositions (which can be considered to be meanings of expressions),

c. expressions.[1]

The notions of the kinds a. and b. are obviously independent on language and precede the notion of the kind c.

Truth of propositions – where possible world propositions are classes of world-time couples – is rather transparent: a proposition is true in a given possible world W at a moment of time T iff its value for this $\langle W, T \rangle$ is the truth-value T. Then, truth of constructions is best definable in terms of truth of propositions constructed by them.[2] Constructions are abstract structured entities akin to algorithms; they construct objects, e.g. propositions. Constructions are 'intensional' entities, thus they can aptly serve for the recently urged hyperintensional individuation of meanings.

The notions of the kind c., truths of expressions, are dependent on, and relative to language(s). The relativity is either explicit, or implicit. Truth of expressions is defined in terms of truth of the expressions' meanings (denotata). Thus unlike the usual approach of Tarski and others, the proposed explication does not depend on the notion of translation (recall that Tarski's method requires that an expression is translated to the theoretician's metalanguage). On the other hand, the present approach relies on the (explicated) notion of language.

It can be shown that the explication resists all forms of the Liar paradox. The explication also confirms Tarski's famous Undefinability theorem, though in a bit supplemented form.

It is also important to stress that the present approach is in some important sense neo-classical. Classical rules, including the Principle of Bi-

[1] Such gradual construction was in fact suggested by Tichý in his remarks on truth (1988, ch. 11 and 12). There, certain (verbal) definitions of the notions can be found. Tichý's investigations surely inspired my approach. The present paper is an extract from a large manuscript on truth; some of my results have been published in (Raclavský 2009).

[2] It is in the spirit of intensional explication of our conceptual scheme to say that propositions can be construed as facts and our world can be construed as a collection of (actual) facts. Then, the proposal of TIL confirms a sort of correspondence theory of truth (true sentences correspond to facts that obtain). However, these issues cannot be discussed here.

valence,[3] are preserved. Because of partiality adopted in the system, however, the rules are appropriately modified. It has a certain connection with the fact that, for each level of truth-notions, there are distinguished total (strong) and partial (weak) variants of the notions.

Employing the truth of non-linguistic entities (propositions and constructions), the approach is immune to well-known arguments of the philosophy of language against 'linguistic' treatment of semantic matters. Moreover, the explication of truth by TIL relies on a hyperintensional (procedural) way of explication of meanings.

TIL is based on λ-calculus accompanied by a particular ramified theory of types. It means that it is a very expressive language within which various axiomatic theories (systems) can be formulated (it is thus not an aim of this paper to state any such particular theory or system, *cf.* also below).

Unfortunately, the lack of space does not enable us to discuss any such matter in greater detail. Moreover, an explication of various particular notions of truth which might come to one's mind cannot be provided here, although the approach is capable of such explication.

The paper is organized as follows. The section "2 Elements of TIL" explains briefly the notion of construction, deduction, type theory, and explication of meanings. The sections "3 Truth of propositions" and "4 Truth of constructions" suggest explications of the two kinds of language-independent notions of truth, which are mentioned in titles. The penultimate section "5 Truth of expressions" begins with an explication of language, which is needed especially for the explication of truth of expressions which are explicitly relative to language. Then, truth of expressions which is implicitly relative to language is explicated and the resistance to the Liar paradox is shown. Finally, the limitation of language and thus also the Undefinability Theorem will be briefly discussed.

2 Elements of TIL

The basic ideas of TIL will best be introduced by the following, partly historical, story. In the late 1960s, Tichý began to utilize Church's simple theory of types (i.e. typed λ-calculus) for logical analysis of natural language. To its basic sorts (atomic types) of *individuals* and *truth-values* (T and F), Tichý added two other sorts – those of *possible worlds* and *moments of times/real*

[3]The *Principle of Bivalence* adopted here reads as: for any proposition P, P has at most one of the two truth-values T and F in a given W and T. In other words, a proposition can be gappy; for instance, the proposition 'The king of France is bald' is gappy in the actual W and present T. (Note that I use double quotation marks for quotation of expressions; single quotation marks are used for indication of propositions and other extralinguistic entities, or, sometimes, for indication of a shift in meaning.)

numbers.[4] Together with some semantic doctrines concerning ways to analyze the meaning of an expression, the framework began to rival the much more popular system of R. Montague. Tichý also soon adopted partiality and he mainly explicated a number of phenomena associated with meaning: modalities, propositional attitudes, intensional transitives, descriptions, temporal adjectives, verb tenses, verb aspects, etc.

The second important feature of TIL are its hyperintensional entities. In early 1970s, Tichý realized that possible world intensions are too coarse-grained to be proper meanings of expressions; rather, one needs structured *hyperintensions*.[5] Two main kinds of λ-terms are usually understood as denoting values of functions or functions as such, but Tichý noticed that they can be also understood as expressing applications of functions to arguments or ways of obtaining functions. On the latter, 'intensional', reading of λ-terms, these stand for constructions, i.e. TIL's hyperintensional entities. Some constructions might also be understood as functions in the older sense, i.e. functions as procedures (rules), which contrasts with the modern notion of function as a mere mapping.

Constructions are procedural entities, akin to algorithmic computations (they are not purely set-theoretical objects). Constructions are language independent; TIL λ-terms serve only to depict constructions (in other words, the formal language of TIL has fixed interpretation). Each object, e.g. a proposition, is constructed by infinitely many equivalent but not identical constructions (constructions thus satisfy intensional principle of individuation). Each construction C is specified by two things: i. the object O constructed by C, ii. the way C constructs, dependently on valuation v, the object O (by means of which subconstructions). Note that constructions are closely connected with objects constructed by them.

For a defence of the notion of construction showing mainly its need, *cf.* especially Tichý's book (1988). For the application of TIL to natural language analysis, see Tichý (2004), Duží et al. (2010), or Raclavský (2009). All these books also include various other applications of TIL. For the rest of the paper, we need to bear in mind at least the following matters concerning semantic scheme, specification of constructions, type theory and deduction (consult the aforementioned books for technical details).

[4] In TIL, possible world *intensions* (i.e. *propositions*, *properties*, *relations-in-intension*, etc.) are total or partial functions from world-time couples to certain entities (viz. truth-values, classes of objects, classes of n-tuples of objects, etc.). Among *non-intensions* one can find in TIL, e.g., the well-known classical truth-functions $\neg$, $\rightarrow$, $\wedge$, and $\vee$, the well-known subclasses of classes of ξ-objects $\exists^{\xi}$ and $\forall^{\xi}$ (for any type ξ; the indication "$^{\xi}$" will be usually suppressed), or the well-known identity relation between ξ-objects, $=^{\xi}$. Constructions are also non-intensions.

[5] One of the notorious arguments for adoption of hyperintensions is that due to intensional analysis, beliefs which are equivalent but non-identical are merged to one. On such use of possible world propositions, an argument that one believes that $1+1=2$ thus one believes Fermat's Last Theorem is wrongly rendered as valid.

In order to explicate meanings of (natural) language, Tichý employed a *semantic scheme* which can be prècised as follows:

an expression E
| *E expresses* (*means*) in L:
a construction C = the *meaning* of E in L
| *C constructs*:
an intension / non-intension = the *denotatum* of E in L

Empirical expressions ("the Pope", "tiger", "It rains in Nice", ...) denote intensions; non-empirical expressions ("not", "3", ...) denote non-intensions. The value of an intension in W at T is the *referent* in L, W and T of an empirical expression. The denotatum in L and referent in L, W and T of a non-empirical expression are construed as identical.

Constructions divide into six kinds according to the ways of their constructing. Let X be any object or construction and $C_{(i)}$ be any construction (of order k):

1. *variable* x_k v-constructs the k-th object (of an appropriate type) of the valuation v;
2. *trivialization* ${}^0X$ v-constructs (for any v) the object X directly, without any change (${}^0X$ takes X and leave it as it is);
3. *single execution* ${}^1X$ v-constructs the object (if any) v-constructed by X;
4. *double execution* ${}^2X$ v-constructs the object (if any) which is v-constructed by the construction (if any) v-constructed by X;
5. *composition* $[C\ \ C_1 \ldots C_n]$ v-constructs the value (if any) of the function F (if any) v-constructed by C on the string of entities $A_1 \ldots A_n$ (if any) v-constructed by $C_1, \ldots, C_n$;
6. *closure* $\lambda x C$ v-constructs (for any v) a function which maps the objects in the range of x to the objects which are v-constructed by C (a very much simplified formulation).

Note that the constructions of the kinds iii.–v. can be abortive in the sense that they v-construct nothing whatsoever, they are *v-improper* constructions. For instance, a composition is v-improper when the partial function v-constructed by C is not defined on the string of entities v-constructed by $C_1, \ldots, C_n$. Two constructions are *v-congruent* iff they v-construct one and the same object or they are both v-improper.

The lack of space does not enable me to repeat here Tichý's whole definition (1988, 66) of his unique *ramified type theory.* In the basis of the hierarchy, there are atomic types. In case of TIL, for instance, these are types of individuals, truth-values, possible worlds, and moments of times. The rest of first-order types cover all total and partial n-ary functions over the objects belonging to the first-order types (i.e. first-order objects). Higher-order types include especially types for constructions. For instance, there is a particular type containing the k-order constructions, i.e. constructions of the k-order objects (for $1 \leq k \leq n$). Moreover, functions from or to constructions are classified by some higher-order types as well. It is readily seen that the hierarchy of entities is very, very rich.[6]

In several of his papers (*cf.* Tichý 2004), Tichý also exposed a *deduction system* for his type theory, thus also for TIL. Its derivation rules are made from sequents whereas sequents are made from so-called matches; matches consist of constructions and (trivializations of) objects v-constructed by them. Sequents and rules are thus not expressions of a formal language. Derivation rules display properties of objects. To illustrate, the derivation rule $\Phi \cup \{^0\mathrm{T} : o_1\} \Rightarrow {}^0\mathrm{T} : o_2 \models \Phi \Rightarrow {}^0\mathrm{T} : [o_1 \; {}^0\!\!\rightarrow o_2]$, where o_1 and o_2 are variables for truth-values, shows that the material conditional $\rightarrow$ returns T for the couple $\langle \mathrm{T}, \mathrm{T} \rangle$.[7]

Constructions and derivation rules can be organized in *derivation systems* (Raclavský, Kuchyňka 2011). Roughly speaking, they are objectual correlates of axiomatic systems. It follows from the very notion of derivation system that no derivation system can be separated from its objectual area. Thus, if one has (say) a property of truth at one's disposal, it is not inevitable for one to build up a particular derivation system to single out which particular object is the truth property in question. A derivation system is rather a tool for proving facts about an object (say the truth property), whereas the facts are implied by features of the object.

3 Truth of propositions

Truth of propositions is a phenomenon dependent on circumstances, i.e. possible worlds and moments of time. For a proposition to be true in W at T is nothing but having T as a value for that world-time couple.

[6] The stratification of entities into such hierarchy is justified by *four Vicious Circle Principles* (Raclavský 2009), each of them being entailed by the *Principle of Specification*: you cannot fully specify an entity by means of the entity itself.

[7] I view *definitions* as certain $\Leftrightarrow$-rules (both $\Rightarrow$ and $\Leftrightarrow$ concern satisfiability of sequents). Two constructions flanking $\Leftrightarrow^{\xi}$ are v-congruent for any v; the type of the object v-constructed by both constructions will be indicated nearby "$\Leftrightarrow$". Definitions can also be viewed as proposing an explication of the intuitive notion whose rigorous correlate occurs in the left hand side of the definition; its right hand side shows in which sense the notion 'is meant', which objects 'fall under' it. *Cf.* (Raclavský 2009).

The notion splits in two variants: the partial and the total one. According to the *partial notion*, a proposition P which is gappy in a given W and T is not true or false. In this case – i.e. when the valuation v assigns such P to the variable p, W to the variable w, and T to the variable t – the construction p_{wt},[8] and thus also $[^0\mathrm{True}^{\pi\mathrm{P}}{}_{wt}\ p]$, is v-improper:

$$[^0\mathrm{True}^{\pi\mathrm{P}}{}_{wt}\ p] \Leftrightarrow^{\mathrm{o}} p_{wt} \text{ (alternatively } [p_{wt}\ {}^0\!=\ {}^0\mathrm{T}]).$$

The extension (a characteristic function) of the property 'True$^{\pi\mathrm{P}}$' in W at T is undefined for P. The definition matches the deflationist intuition that there is a notion of truth which adds nothing to a proposition.

According to the *total notion*, on the other hand, a proposition which is gappy in a given W and T is assigned by the truth-value F (the variable o ranges over the type of truth-values):

$$[^0\mathrm{True}^{\pi\mathrm{T}}{}_{wt}\ p] \Leftrightarrow^{\mathrm{o}} [^0\exists\ \lambda o\ [[o\ {}^0\!= p_{wt}]\ {}^0\!\wedge [o\ {}^0\!=\ {}^0\mathrm{T}]]].$$

In those W and T, a gappy proposition thus falls in the antiextension of the property 'True$^{\pi\mathrm{T}}$'. Every proposition is thus determinately assessed as true or not true.[9]

4 Truth of constructions

The notions of truth of constructions form two groups. In the first group, there are notions of truth *independent on circumstances*. A sample verbal definition: a (k-order) construction is true (or rather: is a *c-truth*) iff it (v-)constructs the truth-value T. In the definition, the variable c^k ranges over the type of k-order constructions and the double execution corresponds to the word "(v-)constructs":

$$[^0\mathrm{Ctruth}^{*k\mathrm{P}} c^k] \Leftrightarrow^{\mathrm{o}} {}^2c^k \text{ (alternatively } [^2c^k\ {}^0\!=\ {}^0\mathrm{T}]).$$

L-truths can be defined as truths v-constructing T on every v.[10]

In the second group, there are kinds of truths of constructions which are *dependent on circumstances*. The sensitivity on circumstances can be traced back to the circumstance sensitivity of truth of propositions which are (v-)constructed by the constructions.

Of course, there is a plenitude of such particular (sub)kinds of truths of constructions because of the plenitude of orders of constructions. (A hier-

[8] "C_{wt}" abbreviates "[[C w] t]".

[9] It is just this notion which should be deployed in appropriate reformulations of classical laws in order to be valid within a framework adopting partiality.

[10] For that purpose a bit richer type basis is needed.

archy.) But there are also various distinct notions of truth of constructions within one particular order, which corresponds to the fact that there are various slightly distinct notions of truth.[11]

For instance, we have both partial and total notions of truth of constructions. In the definition, ${}^{2}c^{k}{}_{wt}$ v-constructs the value (if any) of the proposition (if any) v-constructed by the construction (if any) v-constructed by c^{k}:

$$[{}^{0}\mathrm{True}^{*k\mathrm{P}}{}_{wt}\ c^{k}] \Leftrightarrow^{o}\ [{}^{0}\mathrm{True}^{\pi\mathrm{P}}{}_{wt}\ {}^{2}c^{k}]$$

$$[{}^{0}\mathrm{True}^{*k\mathrm{T}}{}_{wt}\ c^{k}] \Leftrightarrow^{o}\ [{}^{0}\exists\ \lambda o\ [[o\ {}^{0}{=}\ {}^{2}c^{k}{}_{wt}]\ {}^{0}\wedge\ [o\ {}^{0}{=}\ {}^{0}\mathrm{T}]]].$$

But there are even other notions. To give at least one example from a range of several similar notions definable within the framework, let us define a notion according to which constructions of propositions are only determinately assessed as true or not true, while all other constructions (of individuals, of classes of numbers, ...) are left unassessed:

$$[{}^{0}\mathrm{True}^{*k\mathrm{PT}}{}_{wt}\ c^{k}] \Leftrightarrow^{o}\ [{}^{0}\mathrm{True}^{\pi\mathrm{T}}{}_{wt}\ {}^{2}c^{k}].$$

As regards the constructing of the definiens, if ${}^{2}c^{k}$ does not v-construct a proposition, the actual extension of the property 'True$^{\pi\mathrm{PT}}$' cannot be applied, thus the definiens is a v-improper construction (so is the definiendum). The classes (i.e. characteristic functions) which are extensions of the property 'True$^{*k\mathrm{PT}}$' are thus partial.

5 Truth of expressions

Truth of expressions is dependent on language(s).[12] It is thus intuitively correct to say that an expression E is true in L (in W at T) iff it is true (in W at T) what the expression E means in L. My explication matches this natural definition. The truth of expressions is apparently a semantic property or rather relation(-in-intension). In order to explicate the relation, the explication of the notion of language thus has to be undertaken.

Language is a normative system enabling speakers to communicate. It seems sufficient for our purposes to restrict our attention to the expressive, coding aspect of language in the synchronic sense and model it simply as a function from expressions to meanings. In TIL, a *k-order code* L^{k} is

[11] Some of them might be defined also by other theoreticians (assuming here translatability of their results to the present framework).

[12] Truth of expressions' tokens can be defined as dependent on truth of expressions. It is entirely omitted in this paper.

a (partial) function from (Gödelized) expressions to k-order constructions (Tichý 1988, 228).

But language such as English would be better modelled rather as a hierarchy of codes L^1, L^2, ..., L^n (Raclavský 2009). It corresponds to the existence of 'commenting', 'reflective' levels in language – language enables us to comment on its own parts. On such construal, most of everyday communication takes place in the first-order code of the hierarchy; higher-order coding means, which are used for commenting, are not frequently utilized. A particular hierarchy of codes is a class such that i. it involves n codes of n mutually distinct orders, ii. each expression E having a meaning M in L^k has the same meaning M in L^{k+m} (for $1 < m$), and iii. an expression E lacking meaning in L^k can be meaningful in L^{k+m}. One naturally adds also iv. *compositionality* within the codes of the hierarchy.

In consequence of this, every code of a particular hierarchy shares the same expressions as any other code of the same hierarchy; quantification over all of them is unrestricted. Due to order-cumulativity of functions, every k-order code is also a k+1-order code; the type involving n-order codes thus includes nearly all codes of the hierarchy; we can quantify over them. A hierarchy of codes is a certain class; such classes form an n-order type and we can thus quantify over them.

However, every code is limited in its expressive power because no construction of a k-order code L^k is codable in L^k, only in a higher-order code. Moreover, no expression mentioning (precisely: referring to) L^k is endowed with meaning in L^k, only in a higher-order code. To illustrate it, consider the immediate construction of L^k, viz. ${}^0L^k$, which is the meaning of "L^k" (an expression referring to L^k). If ${}^0L^1$ were a value of L^1, L^1 would not be specifiable.

There are two groups of notions of truth of expressions. Let us begin with the first group. Each of its notions determines a relation(-in-intension) between expressions and languages (codes).[13] They are *explicitly language relative notions.*

A prototypical example can be defined as follows, note the perfect match of the definition with the intuitive claim stated in the beginning of this section (e is a variable for numbers/expressions, l^n is a variable for n-order codes):

$$[{}^0\mathrm{TrueIn}^{\mathrm{PT}}{}_{wt}\ e\ l^n] \Leftrightarrow^{\mathrm{o}} [{}^0\mathrm{True}^{*n\mathrm{PT}}{}_{wt}\ [l^n\ e]].$$

The defined notion is such that only expressions denoting (in the respective language) propositions are assessed as true or not true. This is not achieved in the case of the total notion of truth of expressions – its definition

[13]To ask for an expression's meaning in a hierarchy of codes amounts to ask for its meaning in the (virtually) highest code of the hierarchy, i.e. L^n.

is not difficult to come by – which renders also all other expressions as not true.

Note that both the definiendum and the definiens are n+1-order constructions.[14] Hence, they cannot be expressed already in an n-order code (the point can be surely generalized also for $n = 1$). This is the reason why an appropriate version of the Liar paradox is avoided (*cf.* Raclavský 2012).

Each notion from the second group determines a semantic property of expressions, not a relation(-in-intension). Unlike the preceding case, these notions are *implicitly language relative notions* of truth of expressions. Let us consider an example of a concrete definition:[15]

$$[{}^0\mathrm{True}^{\mathrm{L}n\mathrm{T}}{}_{wt}\ e] \Leftrightarrow^{\mathrm{o}} [{}^0\mathrm{TrueIn}^{\mathrm{T}}{}_{wt}\ e\ {}^0\mathrm{L}^n]$$

The definiens removes the ambiguity of the intuitive notion in question. It is thus quite clear that it is L^n rather than $\mathrm{L'}^n$ (belonging to the hierarchy of, say, German), or L^n rather than L^{n-1}, in which the semantic feature of an expression E should be examined. It is perhaps just this ambiguity ubiquitously present in our ordinary and even scientific thinking which is the source of the Liar paradox.

Unlike the definiens, which is an n+1-order construction, the definiendum already is, due to the order-cumulativity of constructions, an n-order construction. It may then seem that a revenge of the Liar paradox is possible. Can be the k-order construction (involving the total notion of truth of expression in L^k)

$$\lambda w \lambda t \lambda e[{}^0\neg\ [{}^0\mathrm{True}^{\mathrm{L}k\mathrm{T}}{}_{wt}\ e]]$$

expressed already in the k-order code L^k?

The Tarskian approach to this question would utilize the appropriate version of the Liar paradox as a *proof* of the negative answer.[16] The negative answer can rely also on the intuitively valid fact – entailed, *inter alia*, by the proof based on the Liar paradox – that *no code with a sufficient expressive power enables us to discuss its own semantic features.*[17]

A very similar fact was concluded already by Tarski (1933/1956, the *Undefinability Theorem*) and also by Tichý (1988, 231, 233). As regards the differences, note that on the present approach the lower-order semantic

[14]The typing technique within Tichý's type theory is similar to that in Russellian ramified type theories.

[15]Recall that if properly closed by lambdas, both constructions flanking $\Leftrightarrow^{\mathrm{o}}$ v-construct one and the same property. It can be proved that the property cannot be discussed by L^n, *cf.* our discussion below.

[16]*Cf.* Tarski (1933) and Tichý (1988, 292-293) or (Raclavský 2012) for such proofs.

[17]In insufficiently expressive codes-languages, a partial truth-predicate can be meaningful without a risk of the Liar paradox (*cf.* Raclavský 2012).

notions (seemingly expressible in object language) are definable, yet they are not expressible in the lower-order codes (object languages). In other words, the proper goal of logicians who investigate truth – viz. to construct a language with a truth-predicate applicable to the expressions of that very language – is not fully achievable if the language in question is sufficiently rich.[18]

6 Conclusions

To stress the essential feature of the TIL approach to truth, the notion of truth is explicated by a three-level system of notions. Truth of expressions is 'supervening', dependent on the lower-level notions of truth which apply to extralinguistic items serving as meanings / denotata of (some) expressions. The TIL approach thus differs significantly from rivalling approaches.

On each level, some novelties with regard to the present understanding are proposed. Truth of possible world propositions is a rather simple notion and it gives rise to no (semantic) paradoxes. Truth of constructions, Tichý's hyperintensional entities, splits into a number of variants along the regulations of the type theory which is governed by a special version of the Vicious Circle Principle. A Liar-like paradox does not ensue because of type restrictions. However, non-paradoxicality is not a primary goal of the implemented type ramification but a product of a reasonable formation of constructions. Truth of expressions depends on truth of constructions they express / propositions they denote. This is dependent, either explicitly or implicitly, on language; thus one has truth as a relation between expressions and languages or/and as a 'relational' property of expressions. The notion of language utilized here results in a hierarchy because of the type hierarchy of constructions. In consequence, the proposal is immune to all forms of the Liar paradox. Recall also that meanings are treated by this approach quite explicitly and that they are explicated as certain hyperintensional entities.

The approach may be seen as a certain 'neo-hierarchical' approach combining Russellian, Tarskian and Kripkean approaches. As regards Russell, however, only some Tichý's constructions roughly appear to be similar to the linguistic entities called 'propositional functions' by Russell. Thus, also the hierarchy of constructions only has an extraneous similarity to Russell's hierarchy of propositional functions. Further, note that Tichý's particular version of the ramified type theory has foundations in simple theory of types. Tarski's hierarchy of languages provides a much better example of a comparable proposal. But the essential difference is that the present hierarchy is based on the hierarchy of constructions which are meanings of expressions, while Tarski did not investigated meanings of the expressions belonging to the languages he considered. The analogy with Kripke can be retained if

[18]Sufficient richness was an original Tarski's condition, *cf.* his (1933).

one ignores some important features of Kripke's proposal, maintaining only that truth comes in total and partial versions.

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